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Three-point correlators of twist-2 operators in N=4 SYM at Born approximation

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Abstract: We calculate two different types of 3-point correlators involving twist-2 operators in the leading weak coupling approximation and all orders in N_c in N=4 SYM theory. Each of three operators in the first correlator can be any component of twist-2 supermultiplet, though the explicit calculation was done for a particular component which is an SU(4) singlet. It is calculated in the leading, Born approximation for arbitrary spins j_1, j_2, j_3. The result significantly simplifies when at least one of the spins is large or equal to zero and the coordinates are restricted to the 2d plane spanned by two light-rays. The second correlator involves two twist-2 operators $\mathrm{Tr}(X\nabla^{j_1}X)+\ldots$, $\mathrm{Tr}(Z\nabla^{j_2}Z)+\ldots$ and one Konishi operator $\mathrm{Tr}[\bar{Z},\bar{X}]^2$. It vanishes in the lowest g^0 order and is computed at the leading g^2 approximation.

Keywords: Supersymmetric gauge theory, AdS-CFT Correspondence, 1/N Expansion, Integrable Field Theories

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1 Introduction

The operator product expansion in N=4 SYM theory, as in any CFT, is completely characterized by its 2-point and 3-point correlators, or, in other words, by the spectrum of anomalous dimensions $\Delta_j(\lambda)$ of local conformal operators $\mathcal{O}_j(x)$ and by the structure constants $C_{ijk}(\lambda)$.[1] Both the dimensions and the structure constants are in general complicated functions of SYM coupling λ and of quantum numbers of the operators. Their calculation is a difficult task generally achievable only within a few orders of Feynman

[1] which are tensors in the general case of operators with spin.

perturbation theory w.r.t. $\lambda = g^2 N$. Things look much better in the planar 't Hooft limit. In this case, not only we can efficiently treat the theory in the large coupling limit, due to the AdS/CFT correspondence, but also there is a strong evidence that the theory is integrable at any coupling λ [1, 2]. This integrability allows, at least in principle and often in practice, efficient calculations of anomalous dimensions at arbitrary λ via the spectral Y-system [2, 3] (or its TBA version [4–6]), either numerically [3, 7–9] or in the strong and weak coupling expansion (up to 3 loops in strong coupling [10] and up to 8 loops in weak coupling in [11–13]).

The situation with the structure constants is more complicated, even in the planar limit. For the moment, we have no closed equations, similar to the spectral AdS/CFT Y-system, defining $C_{ijk}(\lambda)$ for any coupling. One exceptional case is the correlators of BPS-operators which are known at any coupling λ [14–17]. The study for more general operators is limited to the case by case computations in the strong coupling limit [18–25]. In the weak coupling limit, a few interesting particular cases are computed, sometimes up to 3 loops [27–34], and in the SU(2) sector the result is known in general up to one loop [35–37], in a closed form, due to the extensive use of integrability. However, this study shows a presence of some integrable structures giving a hope for general solution of this problem, for any λ and any type of operators. Such solution could be a generalization of the spectral Y-system though it is probably much more complicated than the latter. At present, we have to continue studying the 3-point functions of various sets of operators in various limits, to acquire more of experience in preparation for attacking the general problem.

In the weak coupling limit, a lot can be done in the leading, Born approximation or sometimes even in the next few orders. If the operators are short we don't even need to appeal to the integrability for that. For arbitrary operators, one has to use the integrability to find the right conformal operators (playing the role of Bethe wave functions) and to compute the one-loop graph combinatorics. Most of such results concern the closed scalar SU(2) sector, but the papers [30, 38] contains an interesting example involving twist-2 operators.

In this paper, we calculate two other interesting types of 3-point correlators, and of the corresponding structure constants, involving twist-2 operators. The full supermultiplet of twist-2 operators was constructed in [39]. Our first correlator involves three components of twist-2 super-multiplet with spin j and is calculated in the leading approximation for three such operators of arbitrary spins j_1, j_2, j_3. The result is given by a double integral of three Gegenbauer polynomials, or a triple sum of elementary functions. It appears to be very explicit in the case when at least one spin of j_1, j_2, j_3 is large or zero, which can be compared to the strong coupling computations of [19]. The second correlator involves two twist-2 operators $\mathrm{Tr}(Z\nabla^j Z)$ and one Konishi operator $\mathrm{Tr}[\bar{Z}, \bar{X}]^2$. It appears to be zero in the g^0 order and is computed here at the first non-vanishing g^2 order which is in this case the Born approximation. A computational advantage for this correlator is the absence of mixing of all three operators with other operators at this order.

In conclusions, we discuss a few lessons to be retained from these calculation, for the efforts to guess more general structures of the operator product expansion in N=4 SYM, as well as some future directions.

2 3-point correlator of twist-2 operators in the leading approximation

We start with the computation at the leading order of 3-point correlators of particular twist-2 operators, belonging to the $\mathcal{S}^1_{jl}$ component of twist-2 super-multiplet [39]:

$$\mathcal{S}^1_{jl} = 6\mathcal{O}^{gg}_{jl} + \frac{j}{4}\mathcal{O}^{qq}_{jl} + \frac{j(j+1)}{4}\mathcal{O}^{ss}_{jl}\,. \tag{2.1}$$

The operators

$$\mathcal{O}^{gg}_{jl} = \frac{1}{2}\sigma_j \mathrm{Tr} i^{l-j}(D_{x_2}+D_{x_1})^{l-j}\mathcal{G}^{\frac{5}{2}}_{j-1,x_1,x_2} F^{+\mu}_{\perp}(x_1) g^{\perp}_{\mu\nu} F^{\nu+}_{\perp}(x_2)|_{x_1=x_2}, \tag{2.2}$$

$$\mathcal{O}^{qq}_{jl} = \sigma_j \mathrm{Tr} i^{l-j}(D_{x_2}+D_{x_1})^{l-j}\mathcal{G}^{\frac{3}{2}}_{j,x_1,x_2} \bar\lambda_{\dot\alpha A}\sigma^{+\dot\alpha\beta}(x_1)\lambda^A_\beta(x_2)|_{x_1=x_2}, \tag{2.3}$$

$$\mathcal{O}^{ss}_{jl} = \frac{1}{2}\sigma_j \mathrm{Tr} i^{l-j}(D_{x_2}+D_{x_1})^{l-j}\mathcal{G}^{\frac{1}{2}}_{j+1,x_1,x_2} \bar\phi_{AB}(x_1)\phi^{AB}(x_2)|_{x_1=x_2}, \tag{2.4}$$

realize the highest-weight representation of $\mathrm{SL}(2,\mathcal{R})$ at the g^0 order when $j=l$, and represent its descendants in case of other possible values of l. We have introduced the differential operator $\mathcal{G}^\alpha_{n,x_1,x_2} = i^n(D_{x_2}+D_{x_1})^n C^\alpha_n\left(\frac{D_{x_2}-D_{x_1}}{D_{x_2}+D_{x_1}}\right)$, where $C^\alpha_n(x)$ - Gegenbauer polynomial of order n with index α. D_x are covariant derivatives in the light-like direction n_+: $D_x = n^\mu_+(\partial_\mu - igA_\mu) = \partial_+ - igA_+$ and $\sigma_j = 1-(-1)^j$ (operators at g^0 order are defined only for odd j). Also we will use the operator $\mathcal{G}^{\alpha;g=0}_{n,x_1,x_2}$, which is given by the same expression as $\mathcal{G}^\alpha_{n,x_1,x_2}$, but with the coupling constant $g=0$ which is equivalent to the replacement of covariant derivatives by the ordinary ones. For other notations and definition of fields see appendix A. Acting on a correlation function of three primary operators by derivatives, we can get correlators of any descendants. Thus, we can restrict our attention to the case of three primary operators with $j=l$ without the loss of generality. These superconformal primary operators S^1_{jj} have the one-loop anomalous dimension $\Delta = 3+j+\frac{g^2N_c}{2\pi^2}(\psi(j+2)-\psi(1))$. In g^0 order calculation we should contract only the fields of the same type, and the calculation of correlator $\langle\mathcal{S}^1_{j_1j_1}(x)\mathcal{S}^1_{j_2j_2}(y)\mathcal{S}^1_{j_3j_3}(z)\rangle$ reduces to the calculation of the sum of three independent correlators:

$$\begin{aligned}\langle\mathcal{S}^1_{j_1}(x)\mathcal{S}^1_{j_2}(y)\mathcal{S}^1_{j_3}(z)\rangle &= 6^3\langle\mathcal{O}^{gg}_{j_1}(x)\mathcal{O}^{gg}_{j_2}(y)\mathcal{O}^{gg}_{j_3}(z)\rangle + \frac{j_1j_2j_3}{4^3}\langle\mathcal{O}^{qq}_{j_1}(x)\mathcal{O}^{qq}_{j_2}(y)\mathcal{O}^{qq}_{j_3}(z)\rangle \\ &\quad + \frac{j_1(j_1+1)j_2(j_2+1)j_3(j_3+1)}{4^3}\langle\mathcal{O}^{ss}_{j_1}(x)\mathcal{O}^{ss}_{j_2}(y)\mathcal{O}^{ss}_{j_3}(z)\rangle,\end{aligned} \tag{2.5}$$

where we have introduced short notations $\mathcal{S}^1_j = \mathcal{S}^1_{jj}$ and $\mathcal{O}^{xx}_j = \mathcal{O}^{xx}_{jj}$.

2.1 2 point function

Let us start with the g^0 order calculation of the 2-point function of these twist-2 operators which will be needed anyway for the normalization of their 3-point correlator, in order to extract the corresponding structure constants. The two point correlation function of conformal primary operators $O^{j_1,\ldots,j_k}$ has a fixed tensor structure:

$$\langle O^{\mu_1,\ldots,\mu_k}(x)\bar O^{\nu_1,\ldots,\nu_k}(y)\rangle = C_{O\bar O}\frac{I^{\mu_1\nu_1}\ldots I^{\mu_k\nu_k}}{|x-y|^{2\Delta}}, \tag{2.6}$$

$$I^{\mu\nu} = g^{\mu\nu} - \frac{2(x-y)^\mu(x-y)^\nu}{|x-y|^2}. \tag{2.7}$$

Due to the fact that all indexes are contracted with the light-like vector n_+ in our case, we get the following formula for the operator with k +indices:

$$\langle O_{+,\dots,+}(x)\bar{O}_{+,\dots,+}(y)\rangle = C_{O\bar{O}}\frac{(-2(x-y)_+^2)^k}{|x-y|^{2(\Delta+k)}}. \tag{2.8}$$

It is clear that the restriction of coordinates x and y to 2d plane $\{\perp\}$ does not lead to a loss of generality in case of the two-point correlation functions.

At the leading order, we can factor out all the derivatives from the quantum average, and first calculate the gaussian integrals over fields. It gives

$$\begin{aligned}
&\langle \mathcal{O}^{gg}_{j_1}(x)\mathcal{O}^{gg}_{j_2}(y)\rangle = \\
&= \mathcal{N}^2\frac{\sigma_{j_1}\sigma_{j_2}}{4}\mathcal{G}^{\frac{5}{2};g=0}_{j_1-1,x_1,x_2}\ \mathcal{G}^{\frac{5}{2};g=0}_{j_2-1,y_1,y_2}\langle \mathrm{Tr}\left(F^{+\mu}_{\perp}(x_1)g^{\perp}_{\mu\nu}F^{\nu+}_{\perp}(x_2)\right)\mathrm{Tr}\left(F^{+\mu'}_{\perp}(y_1)g^{\perp}_{\mu'\nu'}F^{\nu'+}_{\perp}(y_2)\right)\rangle|_{\substack{x_1=x_2\\ y_1=y_2}} \\
&= \mathcal{N}^2\sigma_{j_1}\sigma_{j_2}(N_c^2-1)\,\mathcal{G}^{5/2;g=0}_{j_1-1,x_1,x_2}\ \mathcal{G}^{5/2;g=0}_{j_2-1,y_1,y_2}\partial_{x_1+}\partial_{x_2+}\partial_{y_1+}\partial_{y_2+}\frac{1}{|x_1-y_1|^2|x_2-y_2|^2}|_{\substack{x_1=x_2\\ y_1=y_2}},
\end{aligned} \tag{2.9}$$

where in the last equality we used the evenness of Gegenbauer polynomials. The factor $\mathcal{N} = -\frac{1}{8\pi^2}$ is just a normalization for two-point propagators (see appendix A). As it was noticed before, we can put $(x-y)_\perp = 0$. By ordering the points as $x_- > y_-$ and using the formula $\frac{1}{(x-y)_-^k} = \int\limits_0^\infty ds\frac{s^{k-1}}{\Gamma(k)}e^{-s(x-y)_-}$, we can rewrite the action of differential operators as an integral and carry out the calculation (see appendix B). Finally we obtain:

$$\langle \mathcal{O}^{gg}_{j_1}(x)\mathcal{O}^{gg}_{j_2}(y)\rangle_\natural = \delta_{j_1,j_2}\mathcal{N}^2\sigma_{j_1}^2(N_c^2-1)\frac{\Gamma(2j_1+4)\Gamma(j_1+4)}{2^7 3^2\Gamma(j_1)(j_1+3/2)}\frac{1}{(x-y)_+^2(x-y)_-^{2j_1+4}}, \tag{2.10}$$

where the symbol "$\natural$" here and below means the restriction to the plane spanned by $\{n_+, n_-\}$. The full 4-d answer reads as follows:

$$\langle \mathcal{O}^{gg}_{j_1}(x)\mathcal{O}^{gg}_{j_2}(y)\rangle = \delta_{j_1,j_2}\mathcal{N}^2\sigma_{j_1}^2(N_c^2-1)\frac{\Gamma(2j_1+4)\Gamma(j_1+4)2^{2j_1-3}}{3^2\Gamma(j_1)(j_1+3/2)}\frac{(x-y)_+^{2j_1+2}}{((x-y)^2)^{2j_1+4}}. \tag{2.11}$$

Similarly we get the following leading order correlators for the operators $\mathcal{O}^{qq}_j$ and $\mathcal{O}^{ss}_j$:

$$\langle \mathcal{O}^{qq}_{j_1}(x)\mathcal{O}^{qq}_{j_2}(y)\rangle = \delta_{j_1,j_2}\mathcal{N}^2\sigma_{j_1}^2(N_c^2-1)\frac{\Gamma(2j_1+4)\Gamma(j_1+3)2^{2j_1+3}}{\Gamma(j_1+1)(j_1+3/2)}\frac{(x-y)_+^{2j_1+2}}{((x-y)^2)^{2j_1+4}}, \tag{2.12}$$

$$\langle \mathcal{O}^{ss}_{j_1}(x)\mathcal{O}^{ss}_{j_2}(y)\rangle = \delta_{j_1,j_2}\mathcal{N}^2\sigma_{j_1}^2(N_c^2-1)\frac{3\Gamma(2j_1+4)2^{2j_1-1}}{j_1+3/2}\frac{(x-y)_+^{2j_1+2}}{((x-y)^2)^{2j_1+4}}, \tag{2.13}$$

And finally one can assemble the expression for the $\langle \mathcal{S}^1_{j_1}(x)\mathcal{S}^1_{j_2}(y)\rangle$:

$$\langle \mathcal{S}^1_{j_1}(x)\mathcal{S}^1_{j_2}(y)\rangle = \delta_{j_1j_2}\sigma_{j_1}^2(N_c^2-1)H(j_1)\frac{(x-y)_+^{2j_1+2}}{((x-y)^2)^{2j_1+4}}, \tag{2.14}$$

$$H(j_1) = \mathcal{N}^2 j_1(j_1+1)(96+115j_1+35j_1^2)2^{2j_1-4}(2j_1+2)!. \tag{2.15}$$

We will also use the two-point correlator $\langle \mathcal{S}^1_{j_1}(x)\mathcal{S}^1_{j_2}(y)\rangle_\natural$ with coordinates x, y restricted to the $\{n_+, n_-\}$ 2d-plane which is equal to

$$\langle \mathcal{S}^1_{j_1}(x)\mathcal{S}^1_{j_2}(y)\rangle_\natural = \delta_{j_1j_2}H_\natural(j_1)\frac{1}{(x-y)_+^2(x-y)_-^{2j_1+4}},\quad H_\natural(j_1) = \sigma_{j_1}^2(N_c^2-1)\frac{H(j_1)}{2^{2j_1+4}}. \tag{2.16}$$

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Asymptotic form of $H_\natural(j_1)$ when $j_1 \to \infty$ reads as follows:

$$H_\natural(j_1) = \mathcal{N}^2 \sigma_{j_1}^2 (N_c^2-1)\frac{35}{64} j_1^6 \Gamma(2j_1+1)(1+o(j_1^{-1})). \tag{2.17}$$

2.2 3-point function

Let us now proceed with the calculation of the 3-point correlators.

As was noticed before, the correlation function of three components $\mathcal{S}_{j_1}^1(x)$ of twist-2 supermultiplet can be reduced to the sum of correlators of operators with the same fields. Direct use of the Wick rule for the explicit expressions (2.2), (2.3), (2.4) gives

$$\langle \mathcal{O}_{j_1}^{gg}(x)\mathcal{O}_{j_2}^{gg}(y)\mathcal{O}_{j_3}^{gg}(z)\rangle = -2^{10}\mathcal{N}^3(N_c^2-1)\ \mathcal{G}_{j_1-1,x_1,x_2}^{\frac{5}{2};g=0}\ \mathcal{G}_{j_2-1,y_1,y_2}^{\frac{5}{2};g=0}\ \mathcal{G}_{j_3-1,z_1,z_2}^{\frac{5}{2};g=0}\ \Upsilon_2|_\clubsuit, \tag{2.18}$$

$$\langle \mathcal{O}_{j_1}^{qq}(x)\mathcal{O}_{j_2}^{qq}(y)\mathcal{O}_{j_3}^{qq}(z)\rangle = -i2^{9}\mathcal{N}^3(N_c^2-1)\ \mathcal{G}_{j_1,x_1,x_2}^{\frac{3}{2};g=0}\ \mathcal{G}_{j_2,y_1,y_2}^{\frac{3}{2};g=0}\ \mathcal{G}_{j_3,z_1,z_2}^{\frac{3}{2};g=0}\Upsilon_1|_\clubsuit, \tag{2.19}$$

$$\langle \mathcal{O}_{j_1}^{ss}(x)\mathcal{O}_{j_2}^{ss}(y)\mathcal{O}_{j_3}^{ss}(z) = 2^4 3\mathcal{N}^3(N_c^2-1)\ \mathcal{G}_{j_1+1,x_1,x_2}^{\frac{1}{2};g=0}\ \mathcal{G}_{j_2+1,y_1,y_2}^{\frac{1}{2};g=0}\ \mathcal{G}_{j_3+1,z_1,z_2}^{\frac{1}{2};g=0}\Upsilon_0|_\clubsuit, \tag{2.20}$$

where we have introduced function

$$\Upsilon_k = \sigma_{j_1}\sigma_{j_2}\sigma_{j_3}\frac{(x_1-y_2)_+^k(y_1-z_2)_+^k(z_1-x_2)_+^k}{(|x_1-y_2|^2)^{1+k}(|y_1-z_2|^2)^{1+k}(|z_1-x_2|^2)^{1+k}} \tag{2.21}$$

and a new notation $\clubsuit = \{x_i = x,\ y_i = y,\ z_i = z\}$, for the sake of brevity. Now using the formulas (B.5)–(B.7) one can get explicit expressions for correlators:

$$\begin{aligned}\langle \mathcal{O}_{j_1}^{gg}(x)\mathcal{O}_{j_2}^{gg}(y)\mathcal{O}_{j_3}^{gg}(z)\rangle &= \mathcal{N}_{j_1j_2j_3}2^{-2}3^{-3}j_1(j_1+1)j_2(j_2+1)j_3(j_3+1)\\ &\times\sum_{k_1=0}^{j_1-1}\sum_{k_2=0}^{j_2-1}\sum_{k_3=0}^{j_3-1}\eta_g(k_1,k_2,k_3)\theta(k_1,k_2,k_3),\end{aligned} \tag{2.22}$$

where

$$\begin{aligned}\eta_g(k_1,k_2,k_3) &= \binom{j_1-1}{k_1}\binom{j_1+3}{k_1+2}\binom{j_2-1}{k_2}\binom{j_2+3}{k_2+2}\binom{j_3-1}{k_3}\binom{j_3+3}{k_3+2}\\ &\times(j_1+1-k_1+k_2)!(j_2+1-k_2+k_3)!(j_3+1-k_3+k_1)!\end{aligned}$$

and $\mathcal{N}_{j_1j_2j_3} = \mathcal{N}^3(N_c^2-1)\sigma_{j_1}\sigma_{j_2}\sigma_{j_3}i^{j_1+j_2+j_3+3}2^{j_1+j_2+j_3}$

$$\begin{aligned}\langle \mathcal{O}_{j_1}^{qq}(x)\mathcal{O}_{j_2}^{qq}(y)\mathcal{O}_{j_3}^{qq}(z)\rangle &= \mathcal{N}_{j_1j_2j_3}2^6(j_1+1)(j_2+1)(j_3+1)\\ &\times\sum_{k_1=0}^{j_1}\sum_{k_2=0}^{j_2}\sum_{k_3=0}^{j_3}\eta_q(k_1,k_2,k_3)\theta(k_1,k_2,k_3),\end{aligned} \tag{2.23}$$

where

$$\begin{aligned}\eta_q(k_1,k_2,k_3) &= \binom{j_1}{k_1}\binom{j_1+2}{k_1+1}\binom{j_2}{k_2}\binom{j_2+2}{k_2+1}\binom{j_3}{k_3}\binom{j_3+2}{k_3+1}\\ &\times(j_1+1-k_1+k_2)!(j_2+1-k_2+k_3)!(j_3+1-k_3+k_1)!\,,\end{aligned}$$

$$\langle \mathcal{O}_{j_1}^{ss}(x)\mathcal{O}_{j_2}^{ss}(y)\mathcal{O}_{j_3}^{ss}(z)\rangle = \mathcal{N}_{j_1j_2j_3}2^7 3\sum_{k_1=0}^{j_1+1}\sum_{k_2=0}^{j_2+1}\sum_{k_3=0}^{j_3+1}\eta_s(k_1,k_2,k_3)\theta(k_1,k_2,k_3), \tag{2.24}$$

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where

$$\eta_s(k_1,k_2,k_3) = \binom{j_1+1}{k_1}^2 \binom{j_2+1}{k_2}^2 \binom{j_3+1}{k_3}^2 \times (j_1+1-k_1+k_2)!(j_2+1-k_2+k_3)!(j_3+1-k_3+k_1)!$$

and

$$\theta(k_1,k_2,k_3) = \frac{(x-y)_+^{j_1+1-k_1+k_2}}{(|x-y|^2)^{j_1+2-k_1+k_2}} \frac{(y-z)_+^{j_2+1-k_2+k_3}}{(|y-z|^2)^{j_2+2-k_2+k_3}} \frac{(z-x)_+^{j_3+1-k_3+k_1}}{(|z-x|^2)^{j_3+2-k_3+k_1}}. \tag{2.25}$$

Now one can assemble the correlator $\langle \mathcal{S}^1_{j_1}(x)\mathcal{S}^1_{j_2}(y)\mathcal{S}^1_{j_3}(z)\rangle$ using (2.5).

General three-point correlation function of operators with spins in CFT contains a sum over the different tensor structures, as in (C.1). Using the formulas (2.22)–(2.24) one can obtain the expression for an arbitrary tensor structure. But now let us consider a special case by restricting all 3 coordinates to a special 2d plane $\{n_+, n_-\}$. This collapses all tensor structures to a single one (see (C.12)). In this case the correlator reads as follows

$$\begin{aligned}
&\langle O^{\alpha\alpha}_{j_1}(x)O^{\alpha\alpha}_{j_2}(y)O^{\alpha\alpha}_{j_3}(z)\rangle_\natural = \\
&= \frac{B^\alpha_{\natural j_1 j_2 j_3}}{(x-y)_+(y-z)_+(z-x)_+(x-y)_-^{j_1+j_2-j_3+2}(y-z)_-^{j_2+j_3-j_1+2}(z-x)_-^{j_1+j_3-j_2+2}},
\end{aligned} \tag{2.26}$$

where $\alpha \in \{g,q,s\}$ labels the 3 components of the multiplet of twist-2 operators. Setting, without a loss of generality, $x_- = 1, y_- = 0, z_- = -1$ one can get $B^\alpha_{\natural j_1 j_2 j_3}$ from the above formulas as finite triple sums:

$$\begin{aligned}
B^g_{\natural j_1 j_2 j_3} &= \mathcal{N}_{j_1 j_2 j_3} i^2 2^{-2j_2-6} 3^{-3} j_1(j_1+1)j_2(j_2+1)j_3(j_3+1) \times \\
&\quad \times \sum_{k_1=0}^{j_1-1}\sum_{k_2=0}^{j_2-1}\sum_{k_3=0}^{j_3-1} \frac{\eta_g(k_1,k_2,k_3)}{(-2)^{j_3+2-k_3+k_1}},
\end{aligned} \tag{2.27}$$

$$B^q_{\natural j_1 j_2 j_3} = \mathcal{N}_{j_1 j_2 j_3} i^2 2^{-2j_2+2}(j_1+1)(j_2+1)(j_3+1) \sum_{k_1=0}^{j_1}\sum_{k_2=0}^{j_2}\sum_{k_3=0}^{j_3} \frac{\eta_q(k_1,k_2,k_3)}{(-2)^{j_3+2-k_3+k_1}}, \tag{2.28}$$

$$B^s_{\natural j_1 j_2 j_3} = 3\mathcal{N}_{j_1 j_2 j_3} i^2 2^{-2j_2+3} \sum_{k_1=0}^{j_1+1}\sum_{k_2=0}^{j_2+1}\sum_{k_3=0}^{j_3+1} \frac{\eta_s(k_1,k_2,k_3)}{(-2)^{j_3+2-k_3+k_1}}. \tag{2.29}$$

Using (2.16) we can normalize the 3-point correlator $\langle \mathcal{S}^1_{j_1}(x_1)\mathcal{S}^1_{j_2}(x_2)\mathcal{S}^1_{j_3}(x_3)\rangle_\natural$ using the 2-point correlators as follows

$$C_{\natural j_1 j_2 j_3} = \langle \mathcal{S}^1_{j_1}(x_1)\mathcal{S}^1_{j_2}(x_2)\mathcal{S}^1_{j_3}(x_3)\rangle_\natural \sqrt{\prod_{k=1}^{3} \frac{\langle \mathcal{S}^1_{j_k}(x_{k+1})\mathcal{S}^1_{j_k}(x_{k+2})\rangle_\natural}{\langle \mathcal{S}^1_{j_k}(x_k)\mathcal{S}^1_{j_k}(x_{k+1})\rangle_\natural \langle \mathcal{S}^1_{j_k}(x_k)\mathcal{S}^1_{j_k}(x_{k+2})\rangle_\natural}} \tag{2.30}$$

which finally gives the structure constant of these twist two:

$$C_{\natural j_1 j_2 j_3} = \frac{\sigma_{j_1}\sigma_{j_2}\sigma_{j_3}}{8} \frac{\left(6^3 B^g_{\natural j_1 j_2 j_3} + \frac{j_1 j_2 j_3}{4^3} B^q_{\natural j_1 j_2 j_3} + \frac{j_1(j_1+1)j_2(j_2+1)j_3(j_3+1)}{4^3} B^s_{\natural j_1 j_2 j_3}\right)}{\sqrt{H_\natural(j_1)H_\natural(j_2)H_\natural(j_3)}}. \tag{2.31}$$

Expressions for $B_{\natural j_1 j_2 j_3}$ dramatically simplify in the case when one of spins equals two. For $j_3 = 1$ we get:

$$
\begin{aligned}
B^{g}_{\natural j_1 j_2 1} &= 2\mathcal{N}^3 (N_c^2 - 1)\sigma_{j_1}\sigma_{j_2} i^{j_1+j_2} C^{\frac{5}{2}}_{j_1-1}(1) C^{\frac{5}{2}}_{j_2-1}(1)(j_1+j_2)!, && (2.32)\\
B^{q}_{\natural j_1 j_2 1} &= -2^4 3\mathcal{N}^3 (N_c^2 - 1)\sigma_{j_1}\sigma_{j_2} i^{j_1+j_2} C^{\frac{3}{2}}_{j_1}(1) C^{\frac{3}{2}}_{j_2}(1) \\
&\quad \times (2 + 3(j_1+j_2) + j_1^2 + j_2^2)(j_1+j_2)!, && (2.33)\\
B^{s}_{\natural j_1 j_2 1} &= 2^2 3\mathcal{N}^3 \sigma_{j_1}\sigma_{j_2} i^{j_1+j_2} (N_c^2 - 1)(j_1+j_2)![16 + 30(j_1+j_2) + 19(j_1^2 + j_2^2) \\
&\quad + 6(j_1^3 + j_2^3) + j_1^4 + j_2^4 + 36 j_1 j_2 + 4 j_1^2 j_2^2 + 12(j_1^2 j_2 + j_1 j_2^2)], && (2.34)
\end{aligned}
$$

where $C_n^{\alpha}(1) = \frac{\Gamma(2\alpha+n)}{\Gamma(2\alpha)\Gamma(n+1)}$ is the value of Gegenbauer polynomial at the argument equal 1.

2.3 Asymptotic form of 3-point functions

2.3.1 The case of three large spins.

Another case when expressions (2.27)–(2.29) dramatically simplify is the limit of large spins $j_1 \sim j_2 \sim j_3 \gg 1$. Notice that the case when both spin and twist of two operators are large and third is BPS was investigated in [26] and agreement between weak and strong coupling was found. In our case we can apply the saddle-point approximation. First, using the Euler-Maclaurin formula we approximate the triple sum through the integral. Using Stirling approximation for binomial coefficients in function η we get for (2.27)–(2.29) the following integral representation:

$$
B^{\alpha}_{\natural j_1, j_2, j_3} \simeq \int\int\int dk_1 dk_2 dk_3 f_{\alpha}(j_1, j_2, j_3, k_1, k_2, k_3) e^{S}, \tag{2.35}
$$

where the label α denotes the type of correlator, $f_{\alpha}(j_1, j_2, j_3, k_1, k_2, k_3)$ is a rational function of a finite order in variables k_i and the action S in the exponent is the universal function for all correlators given by

$$
\begin{aligned}
S &= S(j_1, j_2, j_3, k_1, k_2, k_3) \\
&= (-k_1 + k_3)\ln(-2) - 2k_1 \ln k_1 \\
&\quad - 2(j_1 - k_1)\ln(j_1 - k_1) - 2k_2 \ln k_2 - 2(j_2 - k_2)\ln(j_2 - k_2) - 2k_3 \ln k_3 \\
&\quad - 2(j_3 - k_3)\ln(j_3 - k_3) + (j_1 - k_1 + k_2)\ln(j_1 - k_1 + k_2) \\
&\quad + (j_2 - k_2 + k_3)\ln(j_2 - k_2 + k_3) + (j_3 - k_3 + k_1)\ln(j_3 - k_3 + k_1).
\end{aligned} \tag{2.36}
$$

The 3 saddle-point equations $\frac{\partial S}{\partial k_j}$ =0, k=1,2,3, read as follows

$$
\begin{cases}
-\dfrac{(j_1 - k_1)^2 (j_3 - k_3 + k_1)}{2k_1^2 (j_1 - k_1 + k_2)} = 1, \\[2ex]
\dfrac{(j_2 - k_2)^2 (j_1 - k_1 + k_2)}{k_2^2 (j_2 - k_2 + k_3)} = 1, \\[2ex]
-\dfrac{2(j_3 - k_3)^2 (j_2 - k_2 + k_3)}{k_3^2 (j_3 - k_3 + k_1)} = 1.
\end{cases} \tag{2.37}
$$

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The solution rendering the main contribution can be obtained explicitly

$$\begin{cases} k_1 = \dfrac{j_1(j_3-j_1)}{j_1+j_2+j_3}, \\ k_2 = \dfrac{j_2(2j_1+j_2)}{2(j_1+j_2+j_3)}, \\ k_3 = \dfrac{j_3(j_2+2j_3)}{j_1+j_2+j_3}. \end{cases} \tag{2.38}$$

The function S, and hence the saddle-point equations (2.37) as well as their solution, (2.38) have an asymmetry which corresponds to our particular choice of coordinates of 3 points $x_- = 1, y_- = 0, z_- = -1$ in the correlator. However, we finally get a fully symmetric answer, as it should be:

$$B^g_{\natural j_1 j_2 j_3} = \frac{1}{3^3 2^9 \sqrt{\pi}} \Lambda(j_1, j_2, j_3)(1 + \mathcal{O}(j_k^{-1})), \tag{2.39}$$

$$B^q_{\natural j_1 j_2 j_3} = -\frac{1}{2\sqrt{\pi}(j_1 j_2 j_3)} \Lambda(j_1, j_2, j_3)(1 + \mathcal{O}(j_k^{-1})), \tag{2.40}$$

$$B^s_{\natural j_1 j_2 j_3} = \frac{3}{\sqrt{\pi}(j_1 j_2 j_3)^2} \Lambda(j_1, j_2, j_3)(1 + \mathcal{O}(j_k^{-1})), \tag{2.41}$$

where $\Lambda(j_1, j_2, j_3) = i^{j_1+j_2+j_3+3} \mathcal{N}^3 (N_c^2 - 1) \sigma_{j_1} \sigma_{j_2} \sigma_{j_3} (j_1 j_2 j_3)^{\frac{3}{2}} (j_1 + j_2 + j_3 + 4)!$.

Using formula (2.5)we get:

$$\langle \mathcal{S}^1_{j_1}(x) \mathcal{S}^1_{j_2}(y) \mathcal{S}^1_{j_3}(z) \rangle_\natural = \frac{7}{2^7 \sqrt{\pi}} \Lambda(j_1, j_2, j_3)(1 + \mathcal{O}(j_k^{-1})). \tag{2.42}$$

And finally, after normalization (2.31) we get the following explicit structure constant of 3 twist-2 operators (for the chosen component of the multiplet) in the limit $j_1 \sim j_2 \sim j_3 \to \infty$:

$$C_{\natural j_1 j_2 j_3} \simeq \sigma_{j_1} \sigma_{j_2} \sigma_{j_3} \frac{1}{\sqrt{N_c^2 - 1}} \frac{i^{j_1+j_2+j_3+3}}{\pi^{\frac{1}{2}} 5^{\frac{3}{2}} 7^{\frac{1}{2}} 2 (j_1 j_2 j_3)^{\frac{3}{2}}} \frac{(j_1 + j_2 + j_3 + 4)!}{\sqrt{(2j_1)!(2j_2)!(2j_3)!}} (1 + \mathcal{O}(j_k^{-1})). \tag{2.43}$$

2.3.2 The case of two large and one arbitrary spin

The similar analysis can be done in the case when two spins are large $j_1 \sim j_2 \gg j_3$. Expressions for $B_{j_1 j_2 j_3}$ in this case read as follows:

$$\begin{aligned} B^g_{\natural j_1 j_2 j_3} &= \mathcal{N}^3 (N_c^2 - 1) i^{j_1+j_2+j_3+3} 2^{-6} 3^{-2} \sigma_{j_1} \sigma_{j_2} \sigma_{j_3} j_1^4 j_2^4 (j_1 - j_2)^{j_3 - 1} \\ &\quad \times C^{\frac{5}{2}}_{j_3 - 1} \left(\frac{j_1^2 + j_2^2}{j_1^2 - j_2^2} \right) (j_1 + j_2)! (1 + \mathcal{O}(j_k^{-1})), \end{aligned} \tag{2.44}$$

$$\begin{aligned} B^q_{\natural j_1 j_2 j_3} &= \mathcal{N}^3 (N_c^2 - 1) i^{j_1+j_2+j_3+1} 2 \sigma_{j_1} \sigma_{j_2} \sigma_{j_3} j_1^2 j_2^2 (j_1 - j_2)^{j_3} \\ &\quad \times C^{\frac{3}{2}}_{j_3} \left(\frac{j_1^2 + j_2^2}{j_1^2 - j_2^2} \right) (j_1 + j_2 + 1)! (1 + \mathcal{O}(j_k^{-1})), \end{aligned} \tag{2.45}$$

$$\begin{aligned} B^s_{\natural j_1 j_2 j_3} &= \mathcal{N}^3 (N_c^2 - 1) i^{j_1+j_2+j_3+3} 6 \sigma_{j_1} \sigma_{j_2} \sigma_{j_3} (j_1 - j_2)^{j_3 + 1} \\ &\quad \times C^{\frac{1}{2}}_{j_3 + 1} \left(\frac{j_1^2 + j_2^2}{j_1^2 - j_2^2} \right) (j_1 + j_2 + 2)! (1 + \mathcal{O}(j_k^{-1})). \end{aligned} \tag{2.46}$$

Substituting it into (2.31) and using the approximation (2.17) for large j_1 and j_2 we obtain the final expression for the structure constants in this limit.

2.3.3 The case of one large and two arbitrary spins

In the case of one large spin $j_1 \gg j_2, j_3$ we get:

$$
\begin{aligned}
B^g_{\natural j_1 j_2 j_3} &= \mathcal{N}^3(N_c^2-1)i^{j_1+j_2+j_3+3}2^{-3}3^{-1}\sigma_{j_1}\sigma_{j_2}\sigma_{j_3} \\
&\quad\times C^{\frac{5}{2}}_{j_2-1}(1)C^{\frac{5}{2}}_{j_3-1}(1)(j_1+j_2+j_3+3)!(1+\mathcal{O}(j_k^{-1})), && (2.47)\\
B^q_{\natural j_1 j_2 j_3} &= \mathcal{N}^3(N_c^2-1)i^{j_1+j_2+j_3+1}4\sigma_{j_1}\sigma_{j_2}\sigma_{j_3} \\
&\quad\times C^{\frac{3}{2}}_{j_2}(1)C^{\frac{3}{2}}_{j_3}(1)(j_1+j_2+j_3+3)!(1+\mathcal{O}(j_k^{-1})), && (2.48)\\
B^q_{\natural j_1 j_2 j_3} &= \mathcal{N}^3(N_c^2-1)i^{j_1+j_2+j_3+3}6\sigma_{j_1}\sigma_{j_2}\sigma_{j_3}(j_1+j_2+j_3+3)!(1+\mathcal{O}(j_k^{-1})). && (2.49)
\end{aligned}
$$

Substituting it into (2.31) and using the approximation (2.17) for large j_1 we obtain the final expression for the structure constants in this limit.

3 Correlation function of two twist-2 and one Konishi operators

In this section we are going to calculate the three-point correlation function of one Konishi operator

$$
\mathcal{O}_K = \mathrm{Tr}[\bar{X},\bar{Z}]^2 = 2\mathrm{Tr}(\bar{X}\bar{Z}\bar{X}\bar{Z}) - 2\mathrm{Tr}(\bar{X}^2\bar{Z}^2) \tag{3.1}
$$

and two scalar twist-2 operators of spins j_1+1 and j_2+1 from the $[O^{ss,\,20}_{jj}]^{CD}_{AB}$ component of twist-2 supermultiplet [39]:

$$
\begin{aligned}
{}[O^{ss,\,20}_{j_1j_1}]^{32}_{41}(\alpha) &= 6\sigma_{j_1}\mathrm{Tr}[\mathcal{G}^{\frac{1}{2}}_{j_1+1,\alpha_1,\alpha_2}X(\alpha_1)X(\alpha_2)]|_{\alpha=\alpha_1=\alpha_2}, \\
{}[O^{ss,\,20}_{j_2j_2}]^{21}_{43}(\alpha) &= 6\sigma_{j_2}\mathrm{Tr}[\mathcal{G}^{\frac{1}{2}}_{j_2+1,\alpha_1,\alpha_2}Z(\alpha_1)Z(\alpha_2)]|_{\alpha=\alpha_1=\alpha_2}.
\end{aligned} \tag{3.2}
$$

Let us note that these operators belong to the irrep **20** w.r.t. the SU(4) and vanish for even j. Minimal spin of such operators is equal to 2, when $j=1$. Let us note that a mixing with double-trace operators could potentially make contribution to such correlators which have only two "sides"[2] in the leading order. But it turns out that both Konishi and twist-2 operators do not have such a mixing. It is true in the case of twist-2 operators as they are constructed only from two scalar fields, and there is no possibility to construct double-trace operators from them. In turn $\mathcal{O}_K$ is a descendent of the lowest component of Konishi supermultiplet and it doesn't mix with other operators [40, 41].

3.1 g^0 calculation

Let us first show that this correlator is zero at the g^0 order. Using the same point-splitting procedure as in the previous section we can rewrite the correlator as follows:

$$
\langle [O^{ss,\,20}_{j_1j_1}]^{32}_{41}(\alpha)[O^{ss,\,20}_{j_2j_2}]^{21}_{43}(\beta)\rangle = \mathcal{K}\langle \mathrm{Tr}\,(X(\alpha_1)X(\alpha_2))\,\mathrm{Tr}\,(Z(\beta_1)Z(\beta_2))\,\mathrm{Tr}[\bar{X},\bar{Z}]^2(\gamma)\rangle|_{\substack{\alpha_{1,2}=\alpha\\ \beta_{1,2}=\beta}}, \tag{3.3}
$$

[2]i.e. the propagators connecting the operators, say, O_1, O_2,O_3 go only between O_1 and O_2 or O_2 and O_3, but not between O_1 and O_3, as in figure 1.

where we have introduced the differential operator $\mathcal{K} = 36\sigma_{j_1}\sigma_{j_2}\ \mathcal{G}^{\frac{1}{2};g=0}_{j_1+1,\alpha_1,\alpha_2}\ \mathcal{G}^{\frac{1}{2};g=0}_{j_2+1,\beta_1,\beta_2}$. The quantum average via Wick rule at this approximation in (3.3) gives (omitting for brevity the coordinate dependence in the l.h.s., obvious from (3.3)):

$$\langle \mathrm{Tr}X^2\mathrm{Tr}Z^2\mathrm{Tr}(\bar{X}\bar{Z}\bar{X}\bar{Z})\rangle = \frac{4\mathcal{N}^4\left(-N_c+\frac{1}{N_c}\right)}{|\alpha_1-\gamma|^2|\alpha_2-\gamma|^2|\beta_1-\gamma|^2|\beta_2-\gamma|^2}, \tag{3.4}$$

$$\langle \mathrm{Tr}X^2\mathrm{Tr}Z^2\mathrm{Tr}(\bar{X}^2\bar{Z}^2)\rangle = \frac{4\mathcal{N}^4\left(N_c^3-2N_c+\frac{1}{N_c}\right)}{|\alpha_1-\gamma|^2|\alpha_2-\gamma|^2|\beta_1-\gamma|^2|\beta_2-\gamma|^2}. \tag{3.5}$$

Now, we can apply the differential operators $\mathcal{G}^{\frac{1}{2};g=0}_{j_1+1,\alpha_1,\alpha_2}$ and $\mathcal{G}^{\frac{1}{2};g=0}_{j_2+1,\beta_1,\beta_2}$ to (3.4) and (3.5), using the relations:

$$(a+b)^n C_n^{\frac{1}{2}}\left(\frac{a-b}{a+b}\right) = \sum_{k=0}^{n} a^k b^{n-k}(-1)^{n-k}\binom{n}{k}^2, \tag{3.6}$$

$$D^k_{x_-}\frac{1}{|x|^2} = D^k_{x_-}\frac{1}{2x_+x_- - x_\perp^2} = \frac{(-1)^k k!(2x_+)^k}{(|x|^2)^{k+1}}. \tag{3.7}$$

The action of $\mathcal{G}^{\frac{1}{2};g=0}_{j_1+1,\alpha_1,\alpha_2}$ on (3.4) or (3.5) gives a factor which vanishes after putting $\alpha_1=\alpha_2=\alpha$:

$$\begin{aligned}
&\lim_{\alpha_{1,2}\to\alpha}(D_{\alpha_1}+D_{\alpha_2})^n C_n^{\frac{1}{2}}\left(\frac{D_{\alpha_1}-D_{\alpha_2}}{D_{\alpha_1}+D_{\alpha_2}}\right)\frac{1}{|\alpha_2-\gamma_3|^2|\alpha_1-\gamma_4|^2} = \\
&= \lim_{\alpha_{1,2}\to\alpha}\sum_{k=0}^{n}(-1)^{n-k}\binom{n}{k}^2\frac{k!(-2(\alpha_{1+}-\gamma_{4+}))^k}{(|\alpha_1-\gamma_4|^2)^{k+1}}\frac{(n-k)!(-2(\alpha_{2+}-\gamma_{3+}))^{n-k}}{(|\alpha_2-\gamma_3|^2)^{n-k+1}} \\
&= \frac{n!(2(\alpha_+-\gamma_+))^n}{(|\alpha-\gamma|^2)^{n+2}}\sum_{k=0}^{n}\binom{n}{k}(-1)^k = 0.
\end{aligned} \tag{3.8}$$

Hence finally we conclude that this correlator vanishes at the at the leading order:

$$\langle [O^{ss,\ 20}_{j_1j_1}]^{32}_{41}(\alpha)[O^{ss,\ 20}_{j_2j_2}]^{21}_{43}(\beta)\mathrm{Tr}[X,Z]^2(\gamma)\rangle|_{g=0} = 0. \tag{3.9}$$

Before proceeding with the g^2 calculation, we write below the two-point correlation functions of two Konishi operators and of two twist-2 operators in the leading order which we will use for the normalization of the 3-point correlator in the leading g^2 approximation:

$$\langle \mathrm{Tr}[X,Z]^2(x)\mathrm{Tr}[\bar{X},\bar{Z}]^2(y)\rangle|_{g=0} = 12\mathcal{N}^4(N_c^4-N_c^2)\frac{1}{|x-y|^8}, \tag{3.10}$$

$$\langle [O^{ss,\ 20}_{j_1j_1}]^{32}_{41}(x)\overline{[O^{ss,\ 20}_{j_1j_1}]^{32}_{41}(y)}\rangle|_{g=0} = \delta_{j_1,j_2}\mathcal{N}^2(N_c^2-1)\sigma^2_{j_1}\Gamma(2j_1+3)3^2 2^{2j_1+5}\frac{(x-y)_+^{2j_1+2}}{((x-y)^2)^{2j_1+4}}, \tag{3.11}$$

$$\langle [O^{ss,\ 20}_{j_1j_1}]^{21}_{43}(x)\overline{[O^{ss,\ 20}_{j_1j_1}]^{21}_{43}(y)}\rangle|_{g=0} = \delta_{j_1,j_2}\mathcal{N}^2(N_c^2-1)\sigma^2_{j_1}\Gamma(2j_1+3)3^2 2^{2j_1+5}\frac{(x-y)_+^{2j_1+2}}{((x-y)^2)^{2j_1+4}}. \tag{3.12}$$

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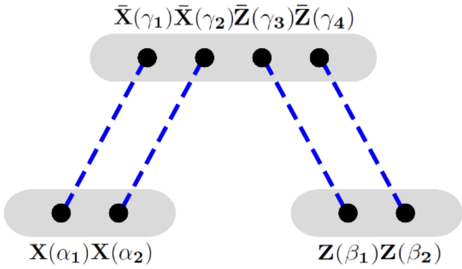

Figure 1. Leading diagram.

3.2 Calculation at g^2 order

Now we will compute this 3-point correlator at g^2 order. The general form of individual terms (coming from the expansion of Gegenbauer polynomials) contributing to the correlator is as follows:

$$\langle \mathrm{Tr}[X,Z]^2(\gamma)\mathrm{Tr}\left(D^{m_1}_{\alpha_1}X(\alpha_1)D^{m_2}_{\alpha_2}X(\alpha_2)\right)\mathrm{Tr}\left(D^{k_1}_{\beta_1}Z(\beta_1)D^{k_2}_{\beta_2}Z(\beta_2)\right)\rangle, \tag{3.13}$$

where the m_1, m_2, k_1, k_2 are some integer powers.

In this case, we use the point-splitting again. Formally, it breaks gauge-invariance and we should insert little Wilson lines between separated points inside the operator. But one can easily check that such Wilson lines do not contribute to the correlator, and can be omitted. All diagrams, which have two free propagators between $Z(X)$ and $\bar{Z}(\bar{X})$, disappear after the action of differential operators, similarly to the g^0 order (3.8). Thus only three types of diagrams contribute. The first one, depicted in figure 2, has two scalar-scalar-gluon vertices, and it looks like a g^0 order diagram with one gluon line connecting two scalar propagators. The second diagram, depicted in figure 3, has one scalar-scalar-gluon vertex connecting a scalar propagator with a gauge field A_+ from covariant derivative in twist-2 operators. And the third diagram , of a type of the one depicted in figure 4, includes the 4-scalar vertex. Let us compute each of these contributions.

3.2.1 Two scalar-scalar-gluon vertices

The structure of scalar-scalar-gluon vertex can be read off from the term $2\mathrm{Tr}D_\mu X D^\mu \bar{X}$ of the N=4 SYM Lagrangian. The relevant terms from this vertex are

$$\begin{aligned} &-2g\mathrm{Tr}([A_\mu, X]\partial^\mu\bar{X} + [A_\mu,\bar{X}]\partial^\mu X) = \\ &= -2g\mathrm{Tr}((\partial^\mu\bar{X}A_\mu X - \bar{X}A_\mu\partial^\mu X) + (\partial^\mu X A_\mu\bar{X} - XA_\mu\partial^\mu\bar{X}))\,. \end{aligned} \tag{3.14}$$

Two relevant vertices, when being put down from the action in the functional integral, give

$$\begin{aligned} W_{\text{ssgss}} = -4g^2\int\!\!\int d^4u d^4v \mathrm{Tr}((\partial^\mu\bar{X}A_\mu X - \bar{X}A_\mu\partial^\mu X) + (\partial^\mu X A_\mu\bar{X} - XA_\mu\partial^\mu\bar{X}))(u) \\ \cdot\mathrm{Tr}((\partial^\nu\bar{Z}A_\nu Z - \bar{Z}A_\nu\partial^\nu Z) + (\partial^\nu Z A_\nu\bar{Z} - ZA_\nu\partial^\nu\bar{Z}))(v). \end{aligned} \tag{3.15}$$

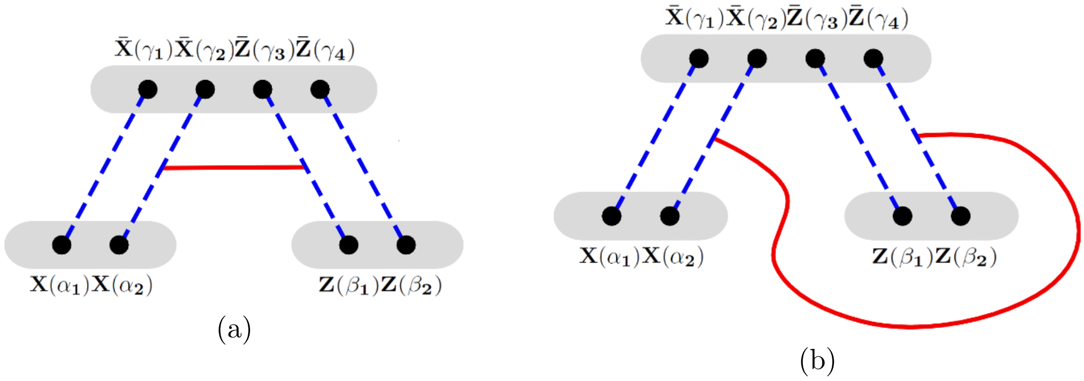

Figure 2. Diagrams with two scalar-scalar-gluon vertex.

Thus for the term $\mathrm{Tr}(\bar{X}^2\bar{Z}^2)$ in Konishi operator we should calculate

$$\langle \mathrm{Tr}(X(\alpha_1)X(\alpha_2))\mathrm{Tr}(Z(\beta_1)Z(\beta_2))\mathrm{Tr}(\bar{X}(\gamma_1)\bar{X}(\gamma_2)\bar{Z}(\gamma_3)\bar{Z}(\gamma_4))W_{\text{ssgss}}\rangle. \tag{3.16}$$

Let us consider the diagram depicted on figure 2.a. From (3.15) we get four terms of the type $\mathrm{Tr}[\partial^\mu\bar{X}A_\mu X - \bar{X}A_\mu\partial^\mu X]\mathrm{Tr}[(\partial^\nu\bar{Z}A_\nu Z - \bar{Z}A_\nu\partial^\nu Z)]$ and the corresponding expressions read as follows[3]

$$(-4g^2)\left(-2N_c^2+6-\frac{4}{N_c^2}\right)\frac{\mathcal{N}^7}{|\gamma_1-\alpha_1|^2|\gamma_4-\beta_2|^2}V(\gamma_2,\gamma_3,\alpha_2,\beta_1)+\left(\begin{smallmatrix}\alpha_1\leftrightarrow\alpha_2,\\ \beta_1\leftrightarrow\beta_2\end{smallmatrix}\right), \tag{3.17}$$

$$-(-4g^2)\left(N_c^4-5N_c^2+8-\frac{4}{N_c^2}\right)\frac{\mathcal{N}^7}{|\gamma_1-\alpha_1|^2|\gamma_4-\beta_2|^2}V(\gamma_2,\gamma_3,\alpha_2,\beta_1)+\left(\begin{smallmatrix}\alpha_1\leftrightarrow\alpha_2,\\ \beta_1\leftrightarrow\beta_2\end{smallmatrix}\right), \tag{3.18}$$

$$-(-4g^2)\left(4-\frac{4}{N_c^2}\right)\frac{\mathcal{N}^7}{|\gamma_1-\alpha_1|^2|\gamma_4-\beta_2|^2}V(\gamma_2,\gamma_3,\alpha_2,\beta_1)+\left(\begin{smallmatrix}\alpha_1\leftrightarrow\alpha_2,\\ \beta_1\leftrightarrow\beta_2\end{smallmatrix}\right), \tag{3.19}$$

$$(-4g^2)\left(-2N_c^2+6-\frac{4}{N_c^2}\right)\frac{\mathcal{N}^7}{|\gamma_1-\alpha_1|^2|\gamma_4-\beta_2|^2}V(\gamma_2,\gamma_3,\alpha_2,\beta_1)+\left(\begin{smallmatrix}\alpha_1\leftrightarrow\alpha_2,\\ \beta_1\leftrightarrow\beta_2\end{smallmatrix}\right). \tag{3.20}$$

where we have introduced the function

$$\begin{aligned}&V(x_1,x_2,x_3,x_4)=\\&=(\partial_3-\partial_1)(\partial_2-\partial_4)\int\int d^4u\,d^4v\frac{1}{|x_1-u|^2|x_3-u|^2|u-v|^2|x_2-v|^2|x_4-v|^2}.\end{aligned} \tag{3.21}$$

For figure 2.b. we get

$$(-4g^2)\left(N_c^4-5N_c^2+8-\frac{4}{N_c^2}\right)\frac{\mathcal{N}^7}{|\gamma_1-\alpha_1|^2|\gamma_3-\beta_1|^2}V(\gamma_2,\gamma_4,\alpha_2,\beta_2)+\left(\begin{smallmatrix}\alpha_1\leftrightarrow\alpha_2,\\ \beta_1\leftrightarrow\beta_2\end{smallmatrix}\right), \tag{3.22}$$

$$-(-4g^2)\left(-2N_c^2+6-\frac{4}{N_c^2}\right)\frac{\mathcal{N}^7}{|\gamma_1-\alpha_1|^2|\gamma_3-\beta_1|^2}V(\gamma_2,\gamma_4,\alpha_2,\beta_2)+\left(\begin{smallmatrix}\alpha_1\leftrightarrow\alpha_2,\\ \beta_1\leftrightarrow\beta_2\end{smallmatrix}\right), \tag{3.23}$$

[3]The symbol $\left(\begin{smallmatrix}\alpha_1\leftrightarrow\alpha_2,\\ \beta_1\leftrightarrow\beta_2\end{smallmatrix}\right)$ denotes all the terms obtained from the previous one (including the sign) by all possible permutations, giving 3 extra terms.

$$-(-4g^2)\left(-2N_c^2+6-\frac{4}{N_c^2}\right)\frac{\mathcal{N}^7}{|\gamma_1-\alpha_1|^2|\gamma_3-\beta_1|^2}V(\gamma_2,\gamma_4,\alpha_2,\beta_2)+\left(\begin{smallmatrix}\alpha_1\leftrightarrow\alpha_2,\\ \beta_1\leftrightarrow\beta_2\end{smallmatrix}\right), \quad (3.24)$$

$$(-4g^2)\left(4-\frac{4}{N_c^2}\right)\frac{\mathcal{N}^7}{|\gamma_1-\alpha_1|^2|\gamma_3-\beta_1|^2}V(\gamma_2,\gamma_4,\alpha_2,\beta_2)+\left(\begin{smallmatrix}\alpha_1\leftrightarrow\alpha_2,\\ \beta_1\leftrightarrow\beta_2\end{smallmatrix}\right). \quad (3.25)$$

Each term from figure 2.a. has a partner from figure 2.b with opposite sign and a function V with different arguments. It turns out that the difference between them doesn't lead to nonzero contribution.[4] Let us show it by rewriting the function $V_{Euclid}(x_1,x_2,x_3,x_4)$, which is the analytic continuation of $V(x_1,x_2,x_3,x_4)$, in the following way [42] in Euclidean space:

$$V_{Euclid}(x_1,x_2,x_3,x_4)=(s-r)\frac{\phi(r,s)}{x_{13}^2x_{24}^2}+(r_1-s_1)\frac{\phi(r_1,s_1)}{x_{13}^2x_{24}^2}+(r_2-s_2)\frac{\phi(r_2,s_2)}{x_{13}^2x_{24}^2}$$
$$+(r_3-s_3)\frac{\phi(r_3,s_3)}{x_{13}^2x_{24}^2}+(r_4-s_4)\frac{\phi(r_4,s_4)}{x_{13}^2x_{24}^2}, \quad (3.26)$$

where

$$r=\frac{x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2}, \quad s=\frac{x_{14}^2x_{23}^2}{x_{13}^2x_{24}^2}, \ r_1=\frac{x_{34}^2}{x_{24}^2}, s_1=\frac{x_{23}^2}{x_{24}^2},$$
$$r_2=\frac{x_{34}^2}{x_{13}^2}, \ s_2=\frac{x_{14}^2}{x_{13}^2}, \ r_3=\frac{x_{12}^2}{x_{24}^2}, s_3=\frac{x_{14}^2}{x_{24}^2}, r_4=\frac{x_{12}^2}{x_{13}^2}, s_4=\frac{x_{23}^2}{x_{13}^2}. \quad (3.27)$$

And the function $\phi(r,s)=\int\limits_0^1 dx\frac{-\ln\left(\frac{r}{s}\right)-2\ln x}{s-x(r+s-1)+x^2r}$ [43] has a simple asymptotics when $r\to 0, s\to 1$ (for $x_1\to x_2$)

$$\phi(r,s)=2+\log\frac{1}{r}+\mathcal{O}\left((1-s)\ln r\right). \quad (3.28)$$

In our case we should replace x_1-x_2 in (3.26) by $\gamma_{23}=\gamma_2-\gamma_3$ for (3.17)–(3.20) and $\gamma_{24}=\gamma_2-\gamma_4$ for (3.22)–(3.25). In the limit when all points $\gamma_i\to\gamma$ when $\gamma_{23}=c_{23}\epsilon$, $\gamma_{24}=c_{24}\epsilon$ with the overall scale $\epsilon\to 0$ and fixed c_{23},c_{24}, we observe the cancelation of log-divergent terms and the rest is proportional to the product of 4 propagators $\sim\frac{\log(c_{23}/c_{24})}{|\gamma-\alpha_1|^2|\gamma-\alpha_2|^2|\gamma-\beta_1|^2|\gamma-\beta_2|^2}$. These terms disappear when acting on them by differential operators (acting only on $\alpha_{1,2},\beta_{1,2}$, and not on γ_j), as in the case of g^0 order. For the second term in Konishi operator $\mathrm{Tr}\bar{X}\bar{Z}\bar{X}\bar{Z}$ we have the terms similar to (3.17)–(3.20), (3.22)–(3.25), but they have the same color-factor $\pm\left(-N_c^2+5-\frac{4}{N_c^2}\right)$, and are subject to similar cancelations. Finally, the answer for the sum of diagrams with two scalar-scalar-gluon vertices is equal to zero.

3.2.2 One scalar-scalar-gluon vertex

In this case the scalar-scalar-gluon vertex connects one scalar propagator to a gluon A_+ from covariant derivative, which is symbolically depicted for $Tr\bar{X}^2\bar{Z}^2$ in figure 3. In

[4] In the planar limit this phenomenon was first noticed in [27, 29].

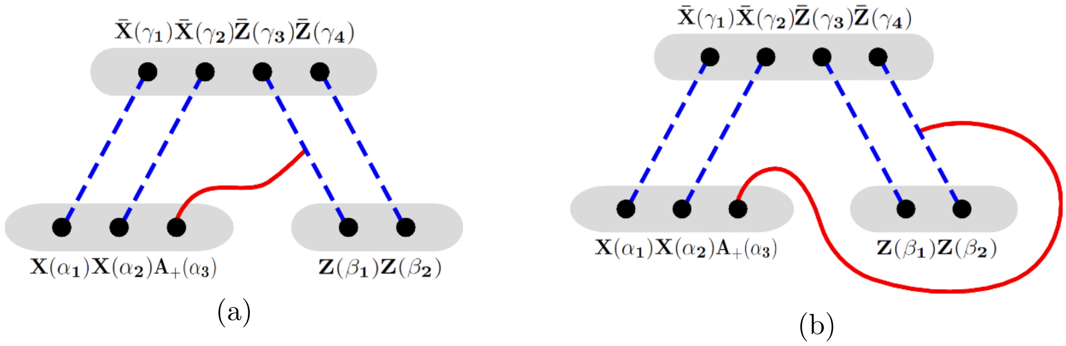

Figure 3. Diagrams with one scalar-scalar-gluon vertex.

this case, we need only the first two terms of expansion over g for $D_+^n X = \partial_+^n X - ig\sum_{k=1}^{n} C_n^k[\partial^{k-1}A_+, \partial^{n-k}X]$, and by factoring out the binomial coefficient and all derivatives, we get $\mathrm{Tr}(XXA_+)$ inside the functional integral. As in the previous case, we get full cancellation of all terms in all orders of N_c, both for $\mathrm{Tr}\bar{X}^2\bar{Z}^2$ and $\mathrm{Tr}\bar{X}\bar{Z}\bar{X}\bar{Z}$ terms of Konishi. For the details see appendix D.

3.2.3 4-scalar vertex and the full g^2 order result

Let us write explicitly the relevant term in the Lagrangian responsible for the 4-scalar interaction (between X and Z components):

$$\frac{1}{8}g^2\mathrm{Tr}[\phi^{AB},\phi^{CD}][\bar{\phi}^{AB},\bar{\phi}^{CD}] = \\ = 2g^2\mathrm{Tr}(2Z\bar{X}\bar{Z}X + 2\bar{X}ZX\bar{Z} - Z\bar{X}X\bar{Z} - \bar{X}Z\bar{Z}X - ZX\bar{X}\bar{Z} - XZ\bar{Z}\bar{X}). \tag{3.29}$$

Carrying out explicit calculation (see appendix D for details) we get the following final expression for this 3-point correlator in the first non-vanishing, Born contribution:

$$K_{j_1j_2}(\alpha,\beta,\gamma) \equiv \langle [O^{ss,\,20}_{j_1j_1}]^{32}_{41}(\alpha)[O^{ss,\,20}_{j_2j_2}]^{21}_{43}(\beta)\mathrm{Tr}[X,Z]^2(\gamma)\rangle \\ = -\sigma_{j_1}\sigma_{j_2}3^3 2^6\pi^2\mathcal{N}^6 g^2(N_c^4-N_c^2)\mathcal{G}^{\frac{1}{2};g=0}_{j_1+1,\alpha_1,\alpha_2}\mathcal{G}^{\frac{1}{2};g=0}_{j_2+1,\beta_1,\beta_2}\Psi|_{\substack{\alpha_{1,2}=\alpha,\\ \beta_{1,2}=\beta}}, \tag{3.30}$$

where Ψ is defined as

$$\Psi = \frac{\log\frac{|\alpha_2-\gamma|^2|\beta_2-\gamma|^2}{|\alpha_2-\beta_2|^2|\epsilon|^2}}{|\alpha_1-\gamma|^2|\alpha_2-\gamma|^2|\beta_1-\gamma|^2|\beta_2-\gamma|^2} + \left(\begin{smallmatrix}\alpha_1\leftrightarrow\alpha_2,\\ \beta_1\leftrightarrow\beta_2\end{smallmatrix}\right). \tag{3.31}$$

This expression looks somewhat similar to the g^0 order expressions for three twist-2 correlators. It is given by the action of differential operators defining the form of the primary operators by the action on a simple function. The latter is just a product of free propagators in the g^0 order calculation, and in the current case it is expressed by the function Ψ containing an extra log. Moreover, the expression (3.30) doesn't have any log terms as it should be in our case when the three-point correlator is zero in the g^0 order. All logarithms can come only from the series expansion of three-point correlator in powers of anomalous dimensions, but this expansion starts at least from g^2. Technically, one can check this cancellation in (3.30) acting directly by differential operators and summing up all terms which have a logarithm. Due to the cancelation of logarithms by action of any derivative, one should keep only those terms which do not act on logarithm. These terms have the form $\log(\ldots)\mathcal{K}*\frac{1}{|\alpha_1-\gamma|^2|\alpha_2-\gamma|^2|\beta_1-\gamma|^2|\beta_2-\gamma|^2}$ and their sum turns out to be equal zero, by the same reasons as at the g^0 order.

Generically, our correlator contains many terms which correspond to different tensor structures (appendix C) as it was in the case of three twist-2 operators. If we restrict the coordinates of the points to the 2-d plane (n_+, n_-) all tensor structures factorize again into a single one (C.13)

$$K_{\natural j_1 j_2}(\alpha,\beta,\gamma) = \frac{B_{\natural j_1 j_2}}{(\alpha-\gamma)_+^2(\beta-\gamma)_+^2(\alpha-\gamma)_-^{2+j_1-j_2}(\beta-\gamma)_-^{2+j_2-j_1}(\alpha-\beta)_-6^{j_1+j_2+2}}. \tag{3.32}$$

Now, if we send γ_- to infinity its asymptotic form reads as follows

$$K_{\natural j_1 j_2}(\alpha,\beta,\gamma) \simeq \frac{B_{\natural j_1 j_2}}{(\alpha-\gamma)_+^2(\beta-\gamma)_+^2(\gamma)_-^4(\alpha-\beta)_-^{j_1+j_2+2}}. \tag{3.33}$$

On the other hand, this asymptotics can be obtained directly from (3.30). Indeed, the leading power in γ_- corresponds to the maximal powers of $(\alpha\beta)_-$ in denominator and appear when all j_1+j_2+2 derivatives acts on the log. It gives us

$$K_{\natural j_1 j_2}(\alpha,\beta,\gamma) = \sigma_{j_1}\sigma_{j_2}3^3 2^4\pi^2\mathcal{N}^6(N_c^4-N_c^2)g^2\frac{i^{j_1+j_2}\Gamma(j_1+j_2+2)}{(\alpha-\gamma)_+^2(\beta-\gamma)_+^2(\gamma)_-^4(\alpha-\beta)_-^{j_1+j_2+2}}. \tag{3.34}$$

Comparing it with (3.33) we conclude that

$$B_{\natural j_1 j_2} = g^2 i^{j_1+j_2}\sigma_{j_1}\sigma_{j_2}3^3 2^4\pi^2\mathcal{N}^6(N_c^4-N_c^2)\Gamma(j_1+j_2+2). \tag{3.35}$$

Normalizing this 3-point function by the corresponding two-point functions (3.10), (3.11) we finally obtain for the normalized structure constant $C_{\natural j_1 j_2}$ of two twist-2 operators and Konishi operator:

$$C_{\natural j_1 j_2} = g^2\sigma_{j_1}\sigma_{j_2}3^{\frac{1}{2}}2^{-4}\pi^{-2}\frac{N_c}{\sqrt{N_c^2-1}}i^{j_1+j_2}\frac{\Gamma(j_1+j_2+2)}{(\Gamma(2j_1+3)\Gamma(2j_2+3))^{\frac{1}{2}}}\,. \tag{3.36}$$

4 Conclusions

In this paper, we have explicitly calculated in the leading, Born approximation two types of 3-point correlators of the SU(N_c) conformal N=4 SYM theory at finite N_c. One of them involves three twist-2 operators with arbitrary spins and another - two twist-2 operators and one Konishi operator (namely, its component built from scalar fields). The related structure constants show a very rich structure of the operator product expansion of the theory, even at this approximation. The g^0 order result for 3 twist-2 operators for the

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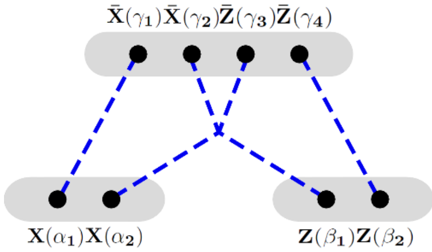

Figure 4. Diagram with a four-scalar vertex.

correlators with arbitrary spins are given by the action of certain differential operators (built from Gegenbauer polynomials) on a standard correlator with the lowest spins. It is interesting that the second correlator, involving Konishi operator and calculated at the leading g^2 approximation, shows the same pattern. This and some other features of our results may provide some insight for the future generalizations to higher orders in the YM coupling and to other operators. Some of the general structures of the OPE at all orders were conjectured from the study of correlators of SU(2) sector involving only scalar fields in [48, 49]. We hope that our attempt to extend calculations to the operators containing covariant derivatives may shed some light on the properties of OPE for the full set of operator of the theory. Of course, if we want integrability to help us on this way we should go to the planar limit. On the other hand, it would be curious to compute, using these structure constants, the multi-point correlators of twist-2 operators in the Born approximation and investigate on this "simple" example some general properties of the full OPE. Another interesting problem to solve is to include higher twist operators, at least at the leading g^0 order.

It would be also interesting to compare our results to the strong coupling calculations of similar correlators (corresponding to the GKP states on the string side) performed in [19]. The details of this comparison contain subtle questions of normalization of operators which have to be elucidated prior to such comparison.

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A Notations

In this section we set our notations. The lagrangian of N=4 SYM with the SU(N_c) gauge group has the following form:

$$\mathfrak{L} = \mathrm{Tr}\left\{ -\frac{1}{2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(D_\mu\phi^{AB})(D^\mu\bar{\phi}_{AB}) + \frac{1}{8}g^2[\phi^{AB},\phi^{CD}][\bar{\phi}_{AB},\bar{\phi}_{CD}] \right.$$
$$\left. +2i\bar{\lambda}_{\dot{\alpha}A}\sigma_\mu^{\dot{\alpha}\beta}D^\mu\lambda_\beta^A - \sqrt{2}g\lambda^{\alpha A}[\bar{\phi}_{AB},\lambda_\alpha^B] + \sqrt{2}g\bar{\lambda}_{\dot{\alpha}A}[\phi^{AB},\bar{\lambda}_B^{\dot{\alpha}}] \right\}, \tag{A.1}$$

where field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$ and covariant derivative $D_\mu = \partial_\mu - ig[A_\mu, \ldots]$.Notice that we work with Minkowski signature $(+,-,-,-)$ and all fields are taken in the adjoint representation of SU(N_c). SO(6)-multiplet with scalars $\phi^a, a \in \{1 \div 6\}$ can be grouped into the antisymmetric tensor ϕ^{AB},$A, B \in \{1 \div 4\}$:

$$\phi^{AB} = \frac{1}{\sqrt{2}}\Sigma^{aAB}\phi^a, \qquad \bar{\phi}_{AB} = \frac{1}{\sqrt{2}}\bar{\Sigma}^a_{AB}\phi^a = (\phi^{AB})^*, \tag{A.2}$$

using Dirac matrices in 6-d Euclidian space:

$$\Sigma^{aAB} = (\eta_{1AB}, \eta_{2AB}, \eta_{3AB}, i\bar{\eta}_{1AB}, i\bar{\eta}_{2AB}, i\bar{\eta}_{3AB}),$$
$$\bar{\Sigma}^a_{AB} = (\eta_{1AB}, \eta_{2AB}, \eta_{3AB}, -i\bar{\eta}_{1AB}, -i\bar{\eta}_{2AB}, -i\bar{\eta}_{3AB}),$$

and 't Hooft symbols:

$$\eta_{iAB} = \epsilon_{iAB} + \delta_{iA}\delta_{4B} - \delta_{iB}\delta_{4A},$$
$$\bar{\eta}_{iAB} = \epsilon_{iAB} - \delta_{iA}\delta_{4B} + \delta_{iB}\delta_{4A},$$

$$\eta_1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \qquad \eta_2 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \qquad \eta_3 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \tag{A.3}$$

$$i\bar{\eta}_1 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}, \qquad i\bar{\eta}_2 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 1 & -i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}, \qquad i\bar{\eta}_3 = \begin{pmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix}. \tag{A.4}$$

Explicit formula for scalars reads as follows

$$[\phi^{AB}] = \frac{1}{\sqrt{2}}(\phi^1\eta_{1AB} + \phi^2\eta_{2AB} + \phi^3\eta_{3AB} + \phi^4 i\bar{\eta}_{1AB} + \phi^5 i\bar{\eta}_{2AB} + \phi^6 i\bar{\eta}_{3AB})$$
$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & \phi^3 + i\phi^6 & -\phi^2 - i\phi^5 & \phi^1 - i\phi^4 \\ -\phi^3 - i\phi^6 & 0 & \phi^1 + i\phi^4 & \phi^2 - i\phi^5 \\ \phi^2 + i\phi^5 & -\phi^1 - i\phi^4 & 0 & \phi^3 - i\phi^6 \\ -\phi^1 + i\phi^4 & -\phi^2 + i\phi^5 & -\phi^3 + i\phi^6 & 0 \end{pmatrix} = \begin{pmatrix} 0 & Z & -Y & \bar{X} \\ -Z & 0 & X & \bar{Y} \\ Y & -X & 0 & \bar{Z} \\ -\bar{X} & -\bar{Y} & -\bar{Z} & 0 \end{pmatrix}.$$

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Fermions are realized as a two-component Weyl spinors λ^A_α with conjugated $\bar\lambda_{\dot\alpha A}$. Spinor index $\alpha \in \{1,2\}$and $A \in \{1 \div 4\}$ is a SU(4) index. Due to supersymmetry one can fix just the propagator of scalars and get the normalization for fermions and gauge fields acting by supercharges. In this article we set the normalization for free propagators as follows:

$$\langle Z(x)^a_b \bar Z(y)^c_d\rangle = \mathcal{N}\left(\delta^a_d\delta^c_b - \frac{1}{N_c}\delta^a_b\delta^c_d\right)\frac{1}{(x-y)^2}, \quad \text{and the same for } X \text{and } Y, \tag{A.5}$$

$$\langle \lambda^A_\alpha(x)^a_b \bar\lambda_{\dot\beta B}(y)^c_d\rangle = i\mathcal{N}\delta^A_B\left(\delta^a_d\delta^c_b - \frac{1}{N_c}\delta^a_b\delta^c_d\right)\bar\sigma^\mu_{\alpha\dot\beta}\frac{\partial}{\partial x^\mu}\frac{1}{(x-y)^2}, \tag{A.6}$$

$$\langle A_\mu(x)^a_b A_\nu(y)^c_d\rangle = -\mathcal{N}\left(\delta^a_d\delta^c_b - \frac{1}{N_c}\delta^a_b\delta^c_d\right)\frac{g_{\mu\nu}}{(x-y)^2}. \tag{A.7}$$

where $\mathcal{N} = -\frac{1}{8\pi^2}$, $\{\sigma^\mu\} = \{1,\sigma\}$ and $\{\bar\sigma^\mu\} = \{1,-\sigma\}$ with ordinary Pauli matrices σ. Throughout the text we use the basis $\{n_+, n_-, e_{1\perp}, e_{2\perp}\}$ with two light-like vectors $n^\mu_+ = \{\frac{1}{\sqrt2}, 0, 0, \frac{1}{\sqrt2}\}$, $n^\mu_- = \{\frac{1}{\sqrt2}, 0, 0, -\frac{1}{\sqrt2}\}$ normalized as $(n_- n_+) = 1$ and two orthogonal vectors $e_{1\perp}, e_{2\perp}$, which span 2-d plane $\{\perp\}$ orthogonal to $\{n_+, n_-\}$. The vector x reads in this basis as $x = x_- n_+ + x_+ n_- + x_\perp$.

A.1 Field content of twist-2 operators

All twist-2 operators, which were discussed in this paper, are constructed from the set of elementary fields $X = \{F^{+\mu}_\perp, \lambda^A_{+\alpha}, \bar\lambda^{\dot\alpha}_{+A}, \phi^{AB}\}$. Twist 2 is the minimal possible twist (defined as dimension minus spin). Gluon field $F^{+\mu}_\perp$ is obtained by projection of one of the indices of the field strength tensor $F^{\mu\nu}$ on n^+ direction where as the second index is automatically restricted to the transverse plane with the metric $g^\perp_{\mu\nu} = g_{\mu\nu} - n_{+\mu}n_{-\nu} - n_{+\nu}n_{-\mu}$. Weyl spinors $\lambda_{+\alpha}$ and $\bar\lambda^{\dot\alpha}_+$ correspond to the states with definite helicity $1,-1$, respectively and they are parameterized as $\lambda_{+\alpha} = \frac12\bar\sigma^-_{\alpha\dot\beta}\sigma^{+\dot\beta\gamma}\lambda_\gamma$ and $\bar\lambda^{\dot\alpha}_+ = \frac12\sigma^{-\dot\alpha\beta}\bar\sigma^+_{\beta\dot\gamma}\bar\lambda^{\dot\gamma}$.

B Twist-2 operators at g^0 order and Gegenbauer polynomials

Explicit formula for conformal twist-2 operators can be obtained from the fact that they are primaries of SL$(2,\mathcal{R})$. It was obtained in [44, 45] and can be expressed in terms of the Jacobi polynomials $P_n^{(2j_1-1,2j_2-1)}(z)$

$$O_n^{j_1,j_2}(x) = X_{j_1}(x) i^n(\overleftarrow{D_+} + \overrightarrow{D_+})^n P_n^{(2j_1-1,2j_2-1)}\left(\frac{\overleftarrow{D_+} - \overrightarrow{D_+}}{\overleftarrow{D_+} + \overrightarrow{D_+}}\right) X_{j_2}(x), \tag{B.1}$$

where j_1, j_2 are the conformal spins and the derivatives $\overleftarrow{D_+}, \overrightarrow{D_+}$ act in light-like direction n_+ on the arguments of the functions $X_{j_1}(x)$ and $X_{j_2}(x)$, respectively.

Gegenbauer polynomials are a particular case of Jacobi polynomials

$$C_n^\alpha(z) = \frac{\Gamma(n+2\alpha)\Gamma(1/2+\alpha)}{\Gamma(2\alpha\Gamma(n+\alpha+1/2))}P_n^{\alpha-\frac12,\alpha-\frac12}(z), \tag{B.2}$$

or, explicitly:

$$C_n^\alpha(z) = \sum_{k=0}^{[\frac{n}{2}]}\frac{(-1)^k(\alpha)_{n-k}(2z)^{n-2k}}{k!(n-2k)!}, \quad (\alpha)_m = \frac{\Gamma(m+\alpha)}{\Gamma(\alpha)}, \tag{B.3}$$

with the orthonormality property

$$\int_{-1}^{1}(1-z^2)^{\alpha-\frac{1}{2}}C_m^\alpha(z)C_n^\alpha(z)dz=\delta_{m,n}\frac{\pi 2^{1-2\alpha}\Gamma(n+2\alpha)}{n!(n+\alpha)\Gamma(\alpha)^2}\ . \tag{B.4}$$

In this paper we have used the following formulae:

$$(b+a)^n C_n^{\frac{1}{2}}\left(\frac{b-a}{b+a}\right)=\sum_{k=0}^{n}(-a)^{n-k}b^k\binom{n}{k}^2, \tag{B.5}$$

$$\begin{aligned}(b+a)^n C_n^{\frac{3}{2}}\left(\frac{b-a}{b+a}\right)&=(n+1)C_n^{\frac{3}{2}}(1)\sum_{k=0}^{n}\frac{a^{n-k}b^k(-1)^{n-k}\binom{n}{k}n!}{(k+1)!(n-k+1)!}\\&=\frac{n+1}{2}\sum_{k=0}^{n}(-a)^{n-k}b^k\binom{n}{k}\binom{n+2}{k+1},\end{aligned} \tag{B.6}$$

$$\begin{aligned}(b+a)^n C_n^{\frac{5}{2}}\left(\frac{b-a}{b+a}\right)&=2(n+1)(n+2)C_n^{\frac{5}{2}}(1)\sum_{k=0}^{n}\frac{a^{n-k}b^k(-1)^{n-k}\binom{n}{k}n!}{(k+2)!(n-k+2)!}\\&=\frac{2(n+1)(n+2)}{4!}\sum_{k=0}^{n}(-a)^{n-k}b^k\binom{n}{k}\binom{n+4}{k+2}\end{aligned} \tag{B.7}$$

which can be proved from the explicit formula (B.3) for definition of Gegenbauer polynomials.

To calculate the 2-point correlator at g^0 order we used integral representations. Say, for gluons it looks as follows

$$\begin{aligned}&\langle\mathcal{O}^{gg}_{j_1}(x)\mathcal{O}^{gg}_{j_2}(y)\rangle=\\&=\sigma_{j_1}\sigma_{j_2}\mathcal{N}^2(N^2-1)\ \mathcal{G}^{\frac{5}{2}}_{j_1-1,x_1,x_2}\ \mathcal{G}^{\frac{5}{2}}_{j_2-1,y_1,y_2}\int_0^\infty\int_0^\infty ds_1ds_2\frac{s_1^2s_2^2e^{-s_1(x_1-y_1)_--s_2(x_2-y_2)_+}}{4(x_1-y_1)_+(x_2-y_2)_+}.\end{aligned} \tag{B.8}$$

Using the evenness of Gegenbauer polynomials, we can rewrite (B.8) as

$$\begin{aligned}&\langle\mathcal{O}^{gg}_{j_1}(x)\mathcal{O}^{gg}_{j_2}(y)\rangle=\\&=\sigma_{j_1}\sigma_{j_2}\mathcal{N}^2(N^2-1)i^{j_1+j_2-2}\\&\times\int_0^\infty\int_0^\infty(s_2+s_1)^{j_1+j_2-2}C^{\frac{5}{2}}_{j_1-1}\left(\frac{s_2-s_1}{s_2+s_1}\right)C^{\frac{5}{2}}_{j_2-1}\left(\frac{s_2-s_1}{s_2+s_1}\right)\frac{e^{-(s_1+s_2)(x-y)_-}}{(x-y)_+^2}\frac{s_1^2s_2^2}{4}\\&=-\frac{\sigma_{j_1}\sigma_{j_2}\mathcal{N}^2(N^2-1)}{4i^{j_1+j_2}(x-y)_+^2}\\&\times\int_0^\infty ds\int_0^1 d\alpha s s^{j_1+j_2+2}C^{\frac{5}{2}}_{j_1-1}(1-2\alpha)C^{\frac{5}{2}}_{j_2-1}(1-2\alpha)\alpha^2(1-\alpha)^2e^{-s(x-y)_-}\\&=\delta_{j_1,j_2}\mathcal{N}^2\frac{\sigma_{j_1}^2(N^2-1)i^{2j_1-2}}{4}\frac{\Gamma(2j_1+4)}{(x-y)_+^2(x-y)_-^{2j_1+4}}\frac{1}{2^5}\frac{\pi 2^{-4}\Gamma(j_1+4)}{\Gamma(j_1)\left(j_1-1+\frac{5}{2}\right)\Gamma^2\left(\frac{5}{2}\right)}\end{aligned}$$

$$= \delta_{j_1,j_2}\mathcal{N}^2 \frac{\sigma_{j_1}^2\Gamma(2j_1+4)\Gamma(j_1+4)}{2^7 3^2\Gamma(j_1)(j_1+3/2)} \frac{1}{(x-y)_+^2(x-y)_-^{2j_1+4}}, \tag{B.9}$$

where in the second line we introduced new variables $s_1 = s\alpha \quad s_2 = s(1-\alpha)$.

Similarly we can get integral representation in case of three-point correlators (2.27)–(2.29):

$$B^g_{j_1j_2j_3} = 2^{-7}b(j_1,j_2,j_3)\int\limits_{-1}^{1}\int\limits_{-1}^{1} d\alpha d\beta(1-\alpha^2)^2(1-\beta^2)^2 C^{\frac{5}{2}}_{j_1-1}(\alpha)C^{\frac{5}{2}}_{j_3-1}(\beta)L^{\frac{5}{2}}_{j_2-1}, \tag{B.10}$$

$$B^q_{j_1j_2j_3} = -2b(j_1,j_2,j_3)\int\limits_{-1}^{1}\int\limits_{-1}^{1} d\alpha d\beta(1-\alpha^2)(1-\beta^2) C^{\frac{3}{2}}_{j_1}(\alpha)C^{\frac{3}{2}}_{j_3}(\beta)L^{\frac{3}{2}}_{j_2}, \tag{B.11}$$

$$B^s_{j_1j_2j_3} = 2^2 b(j_1,j_2,j_3)\int\limits_{-1}^{1}\int\limits_{-1}^{1} d\alpha d\beta C^{\frac{1}{2}}_{j_1+1}(\alpha)C^{\frac{1}{2}}_{j_3+1}(\beta)L^{\frac{1}{2}}_{j_2+1}, \tag{B.12}$$

where

$$b(j_1,j_2,j_3) = i^{j_1+j_2+j_3+3}2^{j_1-j_2+j_3}\mathcal{N}^3(N_c^2-1)\sigma_{j_1}\sigma_{j_2}\sigma_{j_3}(j_1+j_2+j_3+5)!, \tag{B.13}$$

$$L^\kappa_n = C^\kappa_n\left(\frac{1-\alpha\beta}{\alpha-\beta}\right)\frac{(1+\alpha)^{j_3+2}(\alpha-\beta)^n(1+\beta)^{j_1+2}}{(2+\alpha+\beta)^{j_1+j_2+j_3+6}}. \tag{B.14}$$

Due to the fact that $(1-\alpha^2)^{\kappa-\frac{1}{2}}$ is a measure on interval $(-1,1)$ for Gegenbauer polynomials $C^\kappa_n(\alpha)$(B.4) we can interpret expressions (B.10)–(B.12) as an projection of L^κ_n on Gegenbauer polynomials.

C Three-point correlator of operators with spins.

This appendix is a reminder of the formulas obtained in the paper [46, 47], with some precisions for our particular cases. According to its methods, a formula for correlation function of any three primary operators with dimensions Δ_i and spins l_i was obtained, using the embedding formalism. Below we give their expression in original notations and apply it to our particular case of twist-2 operators. Embedding formalism implies the embedding of physical space $\mathcal{V} = \mathcal{R}^d$ ($\mathcal{R}^{d-k,k}$) into the space $\mathcal{M} = \mathcal{R}^{1,d+1}$ ($\mathcal{R}^{d-k+1,k+1}$) where the conformal group $\mathrm{SO}(1,d+1)$ ($\mathrm{SO}(d-k+1,k+1)$) is realized linearly. The vector x from $\mathcal{V}$ lifts up to $\mathcal{M}$ by the formula $x \leftrightarrow P_x = (1,x^2,x)$, which sets the one-to-one correspondence of vectors from $\mathcal{V}$ and light-rays in $\mathcal{M}$. Scalar product of two vectors $P_1 = (P_{1+},P_{1-},p_1)$ and P_2 from $\mathcal{M}$ sets as $(P_1\cdot P_2) = -\frac{P_{1+}P_{2-}+P_{1-}P_{2+}}{2} + p_1p_2$, where p_1p_2 means the scalar product in $\mathcal{V}$. In the paper [46, 47], three vectors of polarization $Z_i \leftrightarrow z_i$ were introduced which contract tensor indices of each operator: $\phi(x,z) = \phi_{a_1,\ldots,a_l}z^{a_1}\ldots z^{a_l}$. In our case this corresponds to the projection of all indexes on n_+ direction as in (2.2)–(2.4). Thus in our case all indices have the same polarization $z_1 = z_2 = z_3 = n_+$. The formula for three-point correlation function reads in these notations as follows:

$$\langle\Phi(P_1,Z_{n_+})\Phi(P_2,Z_{n_+})\Phi(P_3,Z_{n_+})\rangle=\sum_{n_{12},n_{13},n_{23}\geqslant 0}\lambda_{n_{12},n_{13},n_{23}}\begin{bmatrix}\Delta_1&\Delta_2&\Delta_3\\ l_1&l_2&l_3\\ n_{23}&n_{13}&n_{12}\end{bmatrix},\tag{C.1}$$

where summation goes over all possible tensor structures. The coefficients $\lambda_{n_{12},n_{13},n_{23}}$ are labeled by the set $\{n_{12},n_{13},n_{23}\}$ of integers satisfying the following inequalities $m_1=l_1-n_{12}-n_{13}\geqslant 0$, $m_2=l_2-n_{12}-n_{23}\geqslant 0$, $m_3=l_3-n_{13}-n_{23}\geqslant 0$ and the tensor structures are explicitly given by

$$\begin{bmatrix}\Delta_1&\Delta_2&\Delta_3\\ l_1&l_2&l_3\\ n_{23}&n_{13}&n_{12}\end{bmatrix}=\frac{V_1^{m_1}V_2^{m_2}V_3^{m_3}H_{12}^{n_{12}}H_{13}^{n_{13}}H_{23}^{n_{23}}}{P_{12}^{\frac{1}{2}(\tau_1+\tau_2-\tau_3)}P_{13}^{\frac{1}{2}(\tau_1+\tau_3-\tau_2)}P_{23}^{\frac{1}{2}(\tau_2+\tau_3-\tau_1)}},\tag{C.2}$$

where

$$\tau_i=\Delta_i+l_i,\tag{C.3}$$
$$P_{ij}=-2(P_i\cdot P_j)=x_{ij}^2,\tag{C.4}$$
$$H_{ij}=-2\left((Z_i\cdot Z_j)(P_i\cdot P_j)-(Z_i\cdot P_j)(Z_j\cdot P_i)\right)=-2x_{ij+}^2,\tag{C.5}$$
$$V_{i,jk}=\frac{(Z_i\cdot P_j)(P_i\cdot P_k)-(Z_i\cdot P_k)(P_i\cdot P_j)}{(P_j\cdot P_k)},\tag{C.6}$$
$$V_1=V_{1,23}=\frac{x_{21+}x_{13}^2-x_{31+}x_{12}^2}{x_{23}^2},\tag{C.7}$$
$$V_2=V_{2,31}=\frac{x_{32+}x_{21}^2-x_{12+}x_{23}^2}{x_{13}^2},\tag{C.8}$$
$$V_3=V_{3,12}=\frac{x_{13+}x_{23}^2-x_{23+}x_{13}^2}{x_{12}^2}.\tag{C.9}$$

It is easy to see that in the general case all tensor structures are different. In the case when the coordinates are restricted to the $\{n_+,n_-\}$ - plane we get for V much simpler expressions

$$V_1=-\frac{x_{12+}x_{13+}}{x_{23+}},\ V_2=-\frac{x_{23+}x_{12+}}{x_{13+}},\ V_3=-\frac{x_{13+}x_{23+}}{x_{12+}}.\tag{C.10}$$

In the case of twist-2 operators with spin $j+1$ it reduces to

$$\begin{bmatrix}j_1+1&j_2+3&j_3+3\\ j_1+1&j_2+1&j_3+1\\ n_{23}&n_{13}&n_{12}\end{bmatrix}=\frac{(-1)^{j_1+j_2+j_3-n_{12}-n_{13}-n_{23}+3}2^{n_{12}+n_{13}+n_{23}-j_1-j_2-j_3-6}}{x_{12+}x_{13+}x_{23+}x_{12-}^{j_1+j_2-j_3+2}x_{13-}^{j_1+j_3-j_2+2}x_{23-}^{j_2+j_3-j_1+2}}.\tag{C.11}$$

As one can see in this case, all tensor structures have the same coordinate dependence and thus we can rewrite (C.1) as follows:

$$\langle O_{j_1}O_{j_2}O_{j_3}\rangle=\frac{C_{j_1j_2j_3}}{x_{12+}x_{13+}x_{23+}x_{12-}^{j_1+j_2-j_3+2}x_{13-}^{j_1+j_3-j_2+2}x_{23-}^{j_2+j_3-j_1+2}}.\tag{C.12}$$

For the case of two twist-2 operators $O_{j_1}(x_1)$, $O_{j_2}(x_2)$ with spins j_1+1, j_2+1 and one Konishi $O_K(x_3)$ with spin 0 and bare dimension 4 we get

$$\langle O_{j_1}(x_1)O_{j_2}(x_2)O_K(x_3)\rangle = \frac{C_{j_1j_2}}{x_{13+}^2 x_{23+}^2 x_{12-}^{j_1+j_2+2} x_{13-}^{j_1-j_2+2} x_{23-}^{j_2-j_1+2}}. \tag{C.13}$$

D Diagrams at g^2 order

D.1 One scalar-scalar-gluon vertex

Here we present all terms which appear in

$$\langle \mathrm{Tr}(X(\alpha_1)X(\alpha_2)A_+(\alpha_3))\mathrm{Tr}(Z(\beta_1)Z(\beta_2))\mathrm{Tr}(\bar{X}(\gamma_1)\bar{X}(\gamma_2)\bar{Z}(\gamma_3)\bar{Z}(\gamma_4))\Phi_{ssg}\rangle, \tag{D.1}$$

where $\Phi_{ssg} = \int d^4u \mathrm{Tr}((\partial^\mu \bar{X} A_\mu X - \bar{X} A_\mu \partial^\mu X) + (\partial^\mu X A_\mu \bar{X} - X A_\mu \partial^\mu \bar{X}))(u)$ is one scalar-scalar-gluon vertex. The terms corresponding to figure 3.a. read as follows:

$$\int d^4u \partial_{\beta_2+}\left(\left(-2N_c^2+6-\frac{4}{N_c^2}\right)F_{2a1}+\left(N_c^4-5N_c^2+8-\frac{4}{N_c^2}\right)F_{2a2}\right)+(\beta_1\leftrightarrow\beta_2), \tag{D.2}$$

$$-\int d^4u \partial_{\gamma_4+}\left(\left(-2N_c^2+6-\frac{4}{N_c^2}\right)F_{2a1}+\left(N_c^4-5N_c^2+8-\frac{4}{N_c^2}\right)F_{2a2}\right)+(\beta_1\leftrightarrow\beta_2), \tag{D.3}$$

$$\int d^4u \partial_{\gamma_4+}\left(\left(4-\frac{4}{N_c^2}\right)F_{2a1}+\left(-2N_c^2+6-\frac{4}{N_c^2}\right)F_{2a2}\right)+(\beta_1\leftrightarrow\beta_2), \tag{D.4}$$

$$-\int d^4u \partial_{\beta_2+}\left(\left(4-\frac{4}{N_c^2}\right)F_{2a1}+\left(-2N_c^2+6-\frac{4}{N_c^2}\right)F_{2a2}\right)+(\beta_1\leftrightarrow\beta_2). \tag{D.5}$$

where

$$\begin{aligned} F_{2a1} &= F(\alpha_1,\alpha_2,\alpha_3,\beta_1,\beta_2,\gamma_1,\gamma_2,\gamma_3,\gamma_4) \\ &= \frac{1}{|\alpha_3-u|^2|\gamma_3-\beta_1|^2|\gamma_4-u|^2|\beta_2-u|^2|\gamma_1-\alpha_1|^2|\gamma_2-\alpha_2|^2}, \end{aligned} \tag{D.6}$$

$$F_{2a2} = F(\alpha_2,\alpha_1,\alpha_3,\beta_1,\beta_2,\gamma_1,\gamma_2,\gamma_3,\gamma_4). \tag{D.7}$$

For figure 3.b. we get:

$$\int d^4u \partial_{\beta_2+}\left(\left(4-\frac{4}{N_c^2}\right)F_{2b1}+\left(-2N_c^2+6-\frac{4}{N_c^2}\right)F_{2b2}\right)+(\beta_1\leftrightarrow\beta_2), \tag{D.8}$$

$$-\int d^4u \partial_{\gamma_3+}\left(\left(4-\frac{4}{N_c^2}\right)F_{2b1}+\left(-2N_c^2+6-\frac{4}{N_c^2}\right)F_{2b2}\right)+(\beta_1\leftrightarrow\beta_2), \tag{D.9}$$

$$\int d^4u \partial_{\gamma_3+}\left(\left(-2N_c^2+6-\frac{4}{N_c^2}\right)F_{2b1}+\left(N_c^4-5N_c^2+8-\frac{4}{N_c^2}\right)F_{2b2}\right)+(\beta_1\leftrightarrow\beta_2), \tag{D.10}$$

$$-\int d^4u \partial_{\beta_2+}\left((-2N_c^2+6-\frac{4}{N_c^2})F_{2b1}+(N_c^4-5N_c^2+8-\frac{4}{N_c^2})F_{2b2}\right)+(\beta_1\leftrightarrow\beta_2), \tag{D.11}$$

where

$$F_{2b1} = F(\alpha_1,\alpha_2,\alpha_3,\beta_1,\beta_2,\gamma_1,\gamma_2,\gamma_4,\gamma_3), \tag{D.12}$$

$$F_{2b2} = F(\alpha_2,\alpha_1,\alpha_3,\beta_1,\beta_2,\gamma_1,\gamma_2,\gamma_4,\gamma_3). \tag{D.13}$$

Summing up all this terms we get zero in the limit, when $\gamma_i \to \gamma$.

For the second Konishi term $\mathrm{Tr}\bar{X}\bar{Z}\bar{X}\bar{Z}$ we get the same expressions with replacement $\gamma_2 \leftrightarrow \gamma_3$ and the same color factor $\left(-N_c^2+5-\frac{4}{N_c^2}\right)$ for all terms, which again leads to the full cancellation.

D.2 4-scalar vertex

Direct calculation of the contribution from $\mathrm{Tr}2Z\bar{X}\bar{Z}X$ gives us

$$
\begin{aligned}
&\langle \mathrm{Tr}\bar{X}(\gamma_1)\bar{X}(\gamma_2)\bar{Z}(\gamma_3)\bar{Z}(\gamma_4)\mathrm{Tr}X(\alpha_1)X(\alpha_2)\mathrm{Tr}Z(\beta_1)Z(\beta_2)\int d^4u\mathrm{Tr}(2Z\bar{X}\bar{Z}X)(u)\rangle = \\
&= 2\mathcal{N}^6\left(\left(-N_c^2+4-\frac{3}{N_c^2}\right)\frac{T(\gamma_2,\gamma_3,\alpha_2,\beta_2)}{|\gamma_1-\alpha_1|^2|\beta_1-\gamma_4|^2}+\left(-N_c^2+4-\frac{3}{N_c^2}\right)\frac{T(\gamma_1,\gamma_3,\alpha_2,\beta_2)}{|\gamma_2-\alpha_1|^2|\beta_1-\gamma_4|^2}\right.\\
&\quad\left.+\left(-N_c^2+4-\frac{3}{N_c^2}\right)\frac{T(\gamma_2,\gamma_4,\alpha_2,\beta_2)}{|\gamma_1-\alpha_1|^2|\beta_1-\gamma_3|^2}+\left(-N_c^2+4-\frac{3}{N_c^2}\right)\frac{T(\gamma_1,\gamma_4,\alpha_2,\beta_2)}{|\gamma_2-\alpha_1|^2|\beta_1-\gamma_3|^2}\right)\\
&\quad+\left(\begin{smallmatrix}\alpha_1\leftrightarrow\alpha_2,\\ \beta_1\leftrightarrow\beta_2\end{smallmatrix}\right),
\end{aligned}
\tag{D.14}
$$

where we have introduced function

$$
T(\gamma_i,\gamma_j,\alpha,\beta)=\int d^4\omega\frac{1}{|\gamma_i-\omega|^2|\gamma_j-\omega|^2|\alpha-\omega|^2|\beta-\omega|^2}. \tag{D.15}
$$

Euclidian version of this function $T_E=-iT$ has the asymptotic form at $\gamma_i,\gamma_j\to\gamma$ as follows:

$$
T_E(\gamma_i,\gamma_j,\alpha,\beta)\simeq\frac{\pi^2}{|\alpha-\gamma|^2|\beta-\gamma|^2}\left(2+\log\frac{|\alpha-\gamma|^2|\beta-\gamma|^2}{\epsilon_{ij}^2|\alpha-\beta|^2}\right),\quad \epsilon_{ij}=\gamma_i-\gamma_j \tag{D.16}
$$

which can be obtained from the exact expression for (D.15) which was obtained in [43]. Due to this asymptotic form all terms in (D.14) have the same coordinate dependence $\frac{\pi^2\left(2+\log\frac{|\alpha_2-\gamma|^2|\beta_2-\gamma|^2}{|\alpha_2-\beta_2|^2|\epsilon|^2}\right)}{|\alpha_1-\gamma|^2|\alpha_2-\gamma|^2|\beta_1-\gamma|^2|\beta_2-\gamma|^2}$. We can omit the 2 in the numerator because it gives us a term which cancels by action of $\mathcal{G}^{\frac{1}{2};g=0}_{j_1+1,\alpha_1,\alpha_2}\mathcal{G}^{\frac{1}{2};g=0}_{j_2+1,\beta_1,\beta_2}$ as in g^0 case. Moreover , due to the same reason we can retain only the overall scale of ϵ_{ij}'s in (D.16), because the change of this scale only leads to an extra term $\frac{const}{|\alpha_1-\gamma|^2|\alpha_2-\gamma|^2|\beta_1-\gamma|^2|\beta_2-\gamma|^2}$, disappearing after the action of derivatives. Collecting all coefficients we get for the contribution from $\mathrm{Tr}2Z\bar{X}\bar{Z}X$ to the Konishi term $-2\mathrm{Tr}(\bar{X}\bar{X}\bar{Z}\bar{Z})(\gamma)$

$$
4g^2 2\cdot 4\left(-N_c^2+4-\frac{3}{N_c^2}\right)\Omega, \tag{D.17}
$$

where the function Ω is given by

$$
\Omega=\frac{\mathcal{N}^6\pi^2}{|\alpha_1-\gamma|^2|\alpha_2-\gamma|^2|\beta_1-\gamma|^2|\beta_2-\gamma|^2}\log\frac{|\alpha_2-\gamma|^2|\beta_2-\gamma|^2}{|\alpha_2-\beta_2|^2|\epsilon|^2}+\left(\begin{smallmatrix}\alpha_1\leftrightarrow\alpha_2,\\ \beta_1\leftrightarrow\beta_2\end{smallmatrix}\right). \tag{D.18}
$$

For the remaining five terms from (3.29) we get:

$$
4g^2 2\cdot 4\left(-N_c^2+4-\frac{3}{N_c^2}\right)\Omega, \tag{D.19}
$$

$$
-4g^2(2\left(-N_c^2+4-\frac{3}{N_c^2}\right)+\left(N_c^4-4N_c^2+6-\frac{3}{N_c^2}\right)+\left(N_c^2+2-\frac{3}{N_c^2}\right))\Omega, \tag{D.20}
$$

$$
-4g^2(2\left(-N_c^2+4-\frac{3}{N_c^2}\right)+\left(N_c^4-4N_c^2+6-\frac{3}{N_c^2}\right)+\left(N_c^2+2-\frac{3}{N_c^2}\right))\Omega, \tag{D.21}
$$

$$-4g^2(2\left(-N_c^2+4-\frac{3}{N_c^2}\right)+\left(N_c^4-4N_c^2+6-\frac{3}{N_c^2}\right)+\left(N_c^2+2-\frac{3}{N_c^2}\right))\Omega, \tag{D.22}$$

$$-4g^2(2\left(-N_c^2+4-\frac{3}{N_c^2}\right)+\left(N_c^4-4N_c^2+6-\frac{3}{N_c^2}\right)+\left(N_c^2+2-\frac{3}{N_c^2}\right))\Omega. \tag{D.23}$$

Carrying out a similar calculation for the Konishi term $2\mathrm{Tr}\bar{X}\bar{Z}\bar{X}\bar{Z}$ we get:

$$-4g^2 2\cdot 2(2\left(N_c^4-4N_c^2+6-\frac{3}{N_c^2}\right)+2\left(N_c^2+2-\frac{3}{N_c^2}\right))\Omega, \tag{D.24}$$

$$-4g^2 2\cdot 2(2\left(N_c^4-4N_c^2+6-\frac{3}{N_c^2}\right)+2\left(N_c^2+2-\frac{3}{N_c^2}\right))\Omega, \tag{D.25}$$

$$4g^2 4\left(-N_c^2+4-\frac{3}{N_c^2}\right)\Omega, \tag{D.26}$$

$$4g^2 4\left(-N_c^2+4-\frac{3}{N_c^2}\right)\Omega, \tag{D.27}$$

$$4g^2 4\left(-N_c^2+4-\frac{3}{N_c^2}\right)\Omega, \tag{D.28}$$

$$4g^2 4\left(-N_c^2+4-\frac{3}{N_c^2}\right)\Omega. \tag{D.29}$$

And finally, summing up all terms we obtain

$$\begin{aligned}&\langle \mathrm{Tr}X(\alpha_1)X(\alpha_2)\mathrm{Tr}Z(\beta_1)Z(\beta_2)\mathrm{Tr}[\bar{X},\bar{Z}]^2(\gamma)\rangle = \\ &= -48\pi^2 g^2\mathcal{N}^6(N_c^4-N_c^2)\frac{1}{|\alpha_1-\gamma|^2|\alpha_2-\gamma|^2|\beta_1-\gamma|^2|\beta_2-\gamma|^2}\log\frac{|\alpha_2-\gamma|^2|\beta_2-\gamma|^2}{|\alpha_2-\beta_2|^2|\epsilon|^2} \\ &\quad+\left(\begin{smallmatrix}\alpha_1\leftrightarrow\alpha_2\\ \beta_1\leftrightarrow\beta_2\end{smallmatrix}\right).\end{aligned} \tag{D.30}$$

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